Lesson 1

Sets

1.0 Review

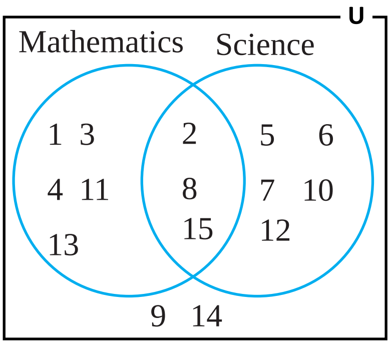
Students from roll no. 1 to 15 in grade 10 are surveyed about whether they like Mathematics or Science. The information from the survey is presented in a Venn-diagram.

Discuss on the following questions based on the Venn-diagram alongside:

a) Write the set of students who like Mathematics by listing method. Write the cardinal number of the set.

b) Write the set of students who like Mathematics only by listing method. Write the cardinal number of the set.

c) Write the set of students who like Science by listing method. Write the cardinal number of the set.



d) Write the set of students who like Science only by listing method. Write the cardinal number of the set.

e) Write the set of students who like both Mathematics and Science by listing method. Write the cardinal number of the set.

f) Write down the set of students who do not like either Mathematics or Science by listing method. Write the cardinal number of the set.

g) How many students were surveyed?

1.1 Cardinality of the Two Sets

Activity 1

An information obtained from the question asked among the students of grade 10 about whether they like coffee or tea. The following information was obtained.

a) The number of students who like coffee is 15.

b) The number of students who like tea is 10.

c) The number of students who like both is 6.

d) The number of students who dislike both is 5.

Discuss the following questions based on the above information:

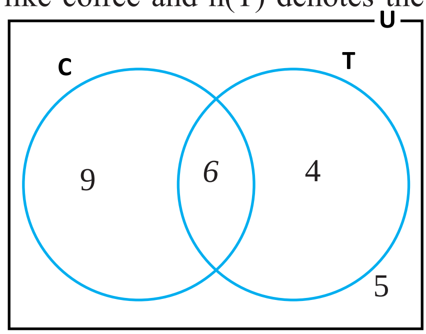
a) How shall the given information be shown in a Venn-diagram?

b) How many students like coffee only?

c) How many students like tea only?

d) How many students were there in the class?

Here, n(C) denotes the number of students who like coffee and n(T) denotes the number of students who like tea. Similarly, $n_0(T)$ and $n_0(C)$ respectively denote the number of students who like tea only and coffee only.



Now, showing the information in a Venn-diagram,

First write the value of $n(C\cap T)$ in venn-diagram the since the number of students who like both is denoted by it.

Then, since the number of students who like coffee is $n_0(C) + n(C\cap T)$,

The number of students who like coffee only $n_0(C) = n(C) - n(C\cap T)$.

i.e. $n_0(C) = 15 - 6 = 9$

Likewise, in case of the number of students who like tea

Let us insert the number of students who like tea only $n_0(T) = n(T) - n(C\cap T)$

$= 10 - 6 = 4.$

Now, let's insert the number of students who dislike tea or coffee $n(\overline{C\cup T}) = 5$

In this way, the total number of students in the class $n(U) = 9 + 6 + 4 + 5 = 24$

If A and B are overlapping sets,

(i) Total number of elements of both the sets, $n(A\cup B) = n(A) + n(B) - n(A\cap B)$.

(ii) The number of elements in the set A only, $n_0(A) = n(A) - n(A\cap B)$.

(iii) The number of elements in the set B only, $n_0(B) = n(B) - n(A\cap B)$.

(iv) Total number of elements of both the sets, $n(A\cup B) = n_0(A) + n_0(B) + n(A\cap B)$.

(v) If A and B are disjoint sets, then $n(A\cup B) = n(A) + n(B)$

(vi) If there are elements of A and B only in U, then $n(U) = n(A\cup B)$

(vii) If there are elements other than A and B in U, then $n(U) = n(A\cup B) + n(\overline{A\cup B})$

Other terminologies:

At least one: $n(A\cup B) = n(A) + n(B) - n(A\cap B)$.

Or, $n(A\cup B) = n_0(A) + n_0(B) + n(A\cap B)$.

At most one: $n(\overline{A\cap B}) = n(U) - n(A\cap B)$.

Exactly one or only one: $n_0(A) + n_0(B) = n(A) + n(B) - 2\times n(A\cap B)$.

Example 1

In a survey of 300 people of a community, it was found that 175 liked cricket and 150 liked football but 25 liked neither of them. Based on this, answer the following questions:

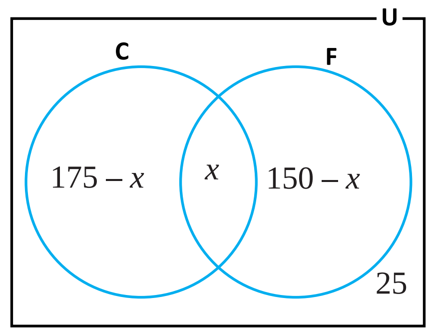
a) Represent the above information in the Venn-diagram.

b) Find the number of people who like both the games.

c) Find the number of people who like exactly one game.

Solution

Let, C and F respectively denote the set of people who like cricket and football. Likewise, U denotes the set of total people.



According to the question,

$n(U) = 300$, $n(C) = 175$, $n(F) = 150$ and $n(\overline{C \cup F}) = 25$

Let, $n(C \cap F) = x$

a) The information is represented in the Venn-diagram alongside.

b) From the Venn-diagram, it can be written as

$$n(U) = n_o(C) + n(C \cap F) + n_o(F) + n(\overline{C \cup F})$$

$$300 = (175 - x) + x + (150 - x) + 25$$

or, $300 = 175 - x + x + 150 - x + 25$

or, $300 = 350 - x$

or, $x = 350 - 300$

$\therefore x = 50$

or, $n(C \cap F) = 50$

$\therefore$ The number of people who like both the games is 50.

Again,

The number of people who liked only cricket, $n_o(C) = 175 - 50 = 125$

The number of people who liked only football, $n_o(F) = 150 - 50 = 100$

$\therefore$ The number of people who liked exactly one game, $n_o(C) + n_o(F) = 125 + 100 = 225$

Example 2

The result of a survey among 120 students of grade 10 is as follows:

30 like only Mathematics.

40 like only English.

10 like neither Mathematics nor English.

Based on this information, answer the following questions:

a) Represent the above information in a Venn-diagram.

b) Find the number of students who like both the subjects.

c) Find the number of students who like at least one subject.

Solution

Let, M and E denote the set of students who like Mathematics and English respectively.

Likewise, U denotes the total students.

According to the question,

$n(U) = 120$, $n_o(M) = 30$, $n_o(E) = 40$ र $n(\overline{M \cup E}) = 10$

Let, $n(M \cap E) = x$

a) The information is represented in a Venn-diagram alongside.

b) From the Venn-diagram,

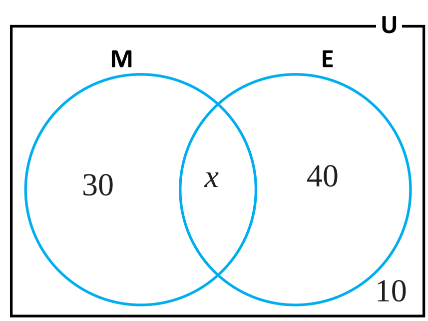
$30 + x + 40 + 10 = 120$

or, $80 + x = 120$

or, $x = 120 - 80 = 40$

$\therefore x = 40$

or, $n(M \cap E) = 40$



$\therefore$ The number of students who like both Mathematics and English is 40.

c) The number of students who like at least one subject $n(M \cup E) = 30 + 40 + 40 = 110$

Example 3

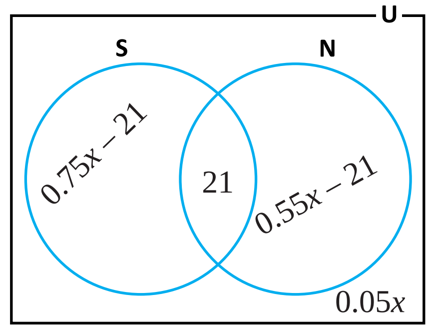
According to a survey among the SEE appeared students from a school, 75% were interested in studying science and 55% were interested in studying staff nurse but 5% denied to give information whilst 21 students were interested to study both science and staff nurse. Based on this information, answer the following questions:

a) Show the above information in the Venn-diagram.

b) Find the total number of students inquired in the survey.

c) Find the number of students who were interested in studying only staff nurse.

Solution



Let, S and N denote the set of students interested to study science and staff nurse respectively. Likewise, U denotes the total students.

According to the question,

Let,

$n(U) = x$,

$n(S) = 75\%$ of $x = 0.75x$,

$n(N) = 55\%$ of $x = 0.55x$,

$n(S \cap N) = 21$ and $n(\overline{S \cup N}) = 5\%$ of $x = 0.05x$,

a) The information is represented in a Venn-diagram alongside.

b) From the Venn-diagram,

$(0.75x - 21) + 21 + (0.55x - 21) + 0.05x = x$

or, $1.35x - 21 = x$

or, $1.35x - x = 21$

or, $0.35x = 21$

or, $x = \frac{21}{0.35} = 60$

$\therefore x = 60$

or, $n(U) = 60$

$\therefore$ The total number of students surveyed was 60.

c) From the Venn-diagram,

The number of students who were interested in studying only staff nurse

$= 0.55x - 21$

$= 0.55 \times 60 - 21$

$= 33 - 21$

$= 12$

$\therefore$ The number of students who were interested in studying only staff nurse was 12.

Example 4

In a survey of 300 foreign tourists visiting to Nepal, it was found that the ratio of the number of tourists who visited Pokhara and Lumbini was 2:3. Among them, 90 visited both the places and 60 visited neither Pokhara nor Lumbini. Based on this information, answer the following questions:

a) Show the above information in a Venn-diagram.

b) Determine the number of tourists who visited only one place.

c) Find the number of tourists who visited at least one of the places.

Solution

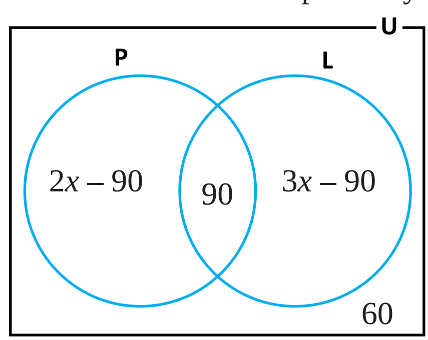
Let, P and L denote the set of tourists who visited Pokhara and Lumbini respectively. Likewise, U denotes the total tourists.

According to the question,

$n(U) = 300$, $n(P \cap L) = 90$ and $n(\overline{P \cup L}) = 60$

Let, $n(P) = 2x$, $n(L) = 3x$

a) The information is represented in a Venn-diagram alongside.



b) From the Venn-diagram,

$(2x - 90) + 90 + (3x - 90) + 60 = 300$

or, $5x - 30 = 300$

or, $5x = 300 + 30$

or, $5x = 330$

or, $x = \frac{330}{5}$

$\therefore x = 66$

Thus, $n_o(P) = 2 \times 66 - 90 = 42$ and $n_o(L) = 3 \times 66 - 90 = 108$

$\therefore$ The number of tourists who visited only one place $= 42 + 108 = 150$

c) The number of tourists who visited at least one place $= 300 - 60 = 240$

Example 5

In a survey among the 200 students studying in grade 10, it was found that the ratio of the number of students who likes Mathematics and English was 2:3. Among them, 30% like both of them but 15% like neither Mathematics nor English. Based on this information, answer the following questions:

a) Represent the above information in a Venn-diagram.

b) What is the difference between the number of students who like Mathematics and the number of students who like English? Find it.

Solution

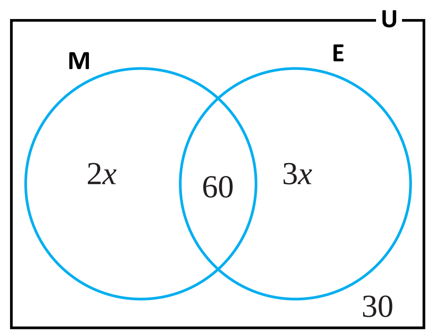
Let, M and E denote the set of students who likes Mathematics and English respectively. Likewise, U denotes the total students.

According to the question,

$n(U) = 200$, $n(M \cap E) = 30\%$ of $200 = 60$ and

$n(\overline{M \cup E}) = 15\%$ of $200 = 30$

Let, $n_o(M) = 2x$, $n_o(E) = 3x$



a) The information is represented in a Venn-diagram alongside.

b) From the Venn-diagram,

$2x + 60 + 3x + 30 = 200$

or, $90 + 5x = 200$

or, $5x = 200 - 90$

or, $5x = 110$

or, $x = \frac{110}{5}$

$\therefore x = 22$

Thus,

The number of students who likes Mathematics, $n(M) = 2x + 60 = 2 \times 22 + 60 = 104$

The number of students who likes English, $n(E) = 3x + 60 = 3 \times 22 + 60 = 126$

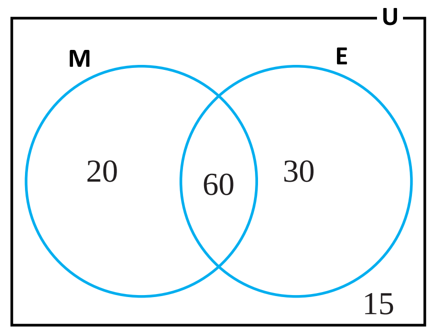
$\therefore$ The difference between the number of students who like Mathematics and the number students who like English = 126 – 104 = 22

Exercise 1.1

1. a) Present the cardinality of sets with examples and show it to your teacher.
   b) For two sets A and B, $A \subset B$, find the values of $n(A \cup B)$ and $n(A \cap B)$.
   c) If A and B are overlapping sets, state the formula for $n(A \cup B)$.
   d) There are 12 and 8 elements in the sets A and B respectively, Find the minimum number of elements that would be in the set $n(A \cup B)$.

2. **In the given Venn-diagram, 80 people are in set M, 90 people are in set E and 15 people are not in both the sets. Determine the cardinality of following sets.**

   (a) $n_o(M)$ (b) $n_o(E)$
   (c) $n(M)$ (d) $n(E)$
   (e) $n(M \cup E)$ (f) $n(M \cap E)$
   (g) $n(\overline{M \cup E})$ (h) $n(U)$



3. a) If $n(U) = 200$, $n_o(M) = 2x$, $n_o(E) = 3x$, $n(M \cap E) = 60$ and $n(\overline{M \cup E}) = 40$ find the value of x.
   b) If $n(U) = 350$, $n(A) = 200$, $n(B) = 220$ and $n(A \cap B) = 120$, then find $n(A \cup B)$ and $n(\overline{A \cup B})$.
   c) If $n(A) = 35$ and $n(\bar{A}) = 25$, then find the value of $n(U)$.
   d) Out of two sets P and Q, there are 40 elements in P, 60 elements in $(P \cup Q)$ and 10 elements in $(P \cap Q)$. How many elements are there in Q? Find.

4. a) **In a survey of 180 students of a school, 45 like Nepali only and 60 like English only but 15 like none of the subjects. Based on this information, answer the following questions:**
      i) Show the above information in a Venn-diagram.
      ii) Find the number of students who like both the subjects.
      iii) Find the number of students who like at least one subject.
   b) **In a survey among the 1200 students of a school, 100 like Mathematics only and 200 like Science only but 700 like neither of the subjects. Based on the information, answer the following questions:**
      i) Show the above information in a Venn-diagram.
      ii) Find the number of students who like both the subjects.

iii) Find the number of students who likes at least one subject.

c) **In a survey among 60 students, 10 play football only and 20 play volleyball only but 12 play neither of the games. Based on the information, answer the following questions:**

i) Show the above information in a Venn-diagram.

ii) Find the number of students who play both the games.

iii) Find the number of students who play at least one game.

5. a) **A survey was carried out among 900 people of a community. According to the survey, 525 read Madhupark, 450 read Yubamanch but 75 didn't read either of the newspapers. Using the information, answer the following questions:**

i) Show the information in a Venn-diagram

ii) Find the number of people who read both the newspapers.

iii) Find the number of people who read only one newspaper.

b) **According to a survey among 150 people, 90 like modern songs, 70 like folk songs but 30 do not like either of the songs. Using the information, answer the following questions:**

i) Show the information in a Venn-diagram

ii) Find the number of people who like both the songs.

iii) Find the number of people who like only modern songs.

c) **According to a survey among 360 players, 210 liked to play volleyball, 180 liked to play football but 30 liked to play neither of the games. Using the information, answer the following questions:**

i) Show the information in a Venn-diagram

ii) Find the number of players who like to play both the games.

iii) Find the number of people who like to play only one game.

6. a) **Out of the students who participated in an examination, 70% passed English, 60% passed Mathematics but 20% failed both the subjects and 550 students passed both the subjects. Based on the information, answer the following questions:**

i) Show the above information in a Venn-diagram.

ii) Find the total number of students participated in the examination.

iii) Find how many students passed English only.

b) **According to a survey of students who have appeared the examination of grade 10, 60% are interested to study Science, 70% are interested to study Management but 10% rejected in the interest to study both Science and Management whilst 400 students are interested to study both Science and Management. Based on this information, answer the following questions:**

   i) Show the above information in a Venn-diagram.

   ii) Find how many students were participated in the survey.

   iii) Find the number of students who are interested to study Science only.

c) **In a survey among people of a community, 65% ride motorcycle, 35% ride scooter but 20% ride both whereas 200 people ride both motorcycle and scooter. Using this information, answer the following questions:**

   i) Show the above information in a Venn-diagram.

   ii) Find how many people were participated in the survey.

   iii) Find the number of people who ride motorcycles only.

7. a) **Among 95 people of a community, it was surveyed that the ratio of the number of people who drink tea and coffee is 4:5, whereas 10 people drink tea but 15 do not drink either tea or coffee. Based on the information, answer the following questions:**

   i) Show the information in a Venn-diagram.

   ii) Find the number of people who drinks exactly one of tea or coffee.

   iii) Find the number of people who drinks at least one; either tea or coffee.

b) **In a survey of 64 students of a class, the ratio of number of students who like milk only and curd only is 2:1 whereas 16 like both based on this information, Answer the following questions:**

i) Show the above information in a Venn-diagram.

   ii) Find the number of students who like milk.

   iii) Find the number of students who like only one kind of drink.

c) **In a conference of 320 participants, it was surveyed that 60 participants only sing and 100 only dance. If the number of people who do not do both is three times the number of people who do both. With the help of this information, answer the following questions:**

i) Show the above information in a Venn-diagram.

ii) Find how many people do not do both genres.

iii) Find the number of people who do one genre at most.

8. **According to a survey of 200 people of a community, it was found that the ratio of the number of people who use laptop only and mobile only is 2:3, among them, 30% use both but 15% does not use both the gadgets. Based on this information, answer the following questions:**

i) Show the above information in a Venn-diagram.

ii) Find the number of people who uses laptop.

iii) Find how many people use one gadget at most.

9. Out of 300 players in a survey, one-third players play volleyball only. 60% of the remaining players play football only. But 60 players do not play both. Then, find the ratio of the number of players who play volleyball and football by using the Venn-diagram.

10. Among 65 players participated in a survey, 11 play volleyball only and 33 play cricket only. If the number of players who play cricket is the double of the number of players who play volleyball, find the number of players who play both and the number of players who does not play both by using Venn-diagram.

11. In a survey of 80 people, 60 like orange and 10 like both orange and apple. The number of people who likes orange is 5 times the number of people who likes apple. By using the Venn-diagram find the number of people who likes apples only and those who do not like both the fruits.

Project Work

Form group of five students each and go to different classes in your schoool. Find answers to the following questions:

Which of the following game do you like? (a) Cricket (b) Football (c) Cricket and football both (d) Others

Upon getting the informations, present it in the Venn-diagram and discuss it in the class.

Answers

1. (a) Show to your teacher. (b) $n(B)$, $n(A)$

   (c) $n(A) + n(B) - n(A \cap B)$ or $n_0(A) + n_0(B) + n(A \cap B)$ (d) 12

2. (a) 20 (b) 30 (c) 80 (d) 90 (e) 110 (f) 60 (g) 15 (h) 125

3. (a) 20 (b) 300, 50 (c) 60 (d) 30

4. (a) (i) 60, (ii) 165 (b) (i) 200 (ii) 500 (c) (i) 18 (ii) 48

5. (a) (i)) 150 (ii) 675 (b) (i) 40 (ii) 50 (c) (i) 60 (ii) 270

6. (a) (i) 1100 (ii) 220 (b) (i) 1000

   (ii) 200 (c) (i) 1000 (ii) 450

7. (a) (i) 70 (ii) 80 (b) ii) 48 (ii) 48

   (c) (i) 120 (ii) 280

8. (i) 104 (ii) 140

9. 6:7

10. 11, 10

11. 4, 6

1.2 Cardinality of Three Sets

Activity 2

The elements of three sets A, B and C are shown in the following two Venn-diagrams. Based on this, discuss the following questions:

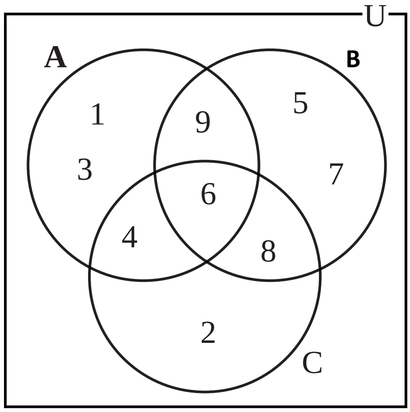


Figure 1

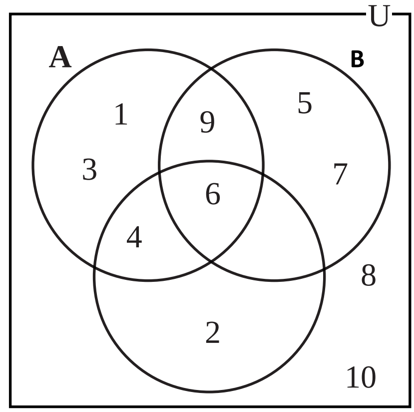


Figure 2

Observing the figure no. 1 and figure no. 2, write the elements of the sets A, B and C by listing method.

What are the cardinalities of each sets A, B and C in figure no. 1?
What are the cardinalities of each sets A, B and C in figure no. 2?
What are the values of n(A∪B∪C) and n(U) in figure no. 1?
What are the values of n(A∪B∪C) and n(U) in figure no. 2?

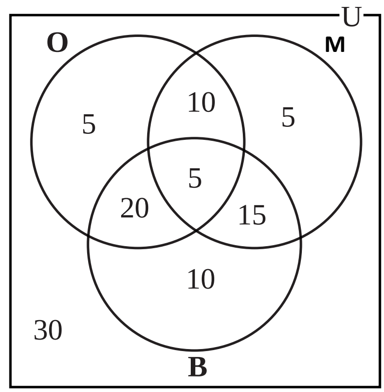
The conclusion from the discussion with friends can be shown below:

Figure 1	**Figure 2**
A = {1, 3, 4, 6, 9} ∴ *n*(A) = 5	A = {1, 3, 4, 6, 9} ∴ *n*(A) = 5
B = {5, 6, 7, 8, 9} ∴ *n*(B) = 5	B = {5, 6, 7, 9} ∴ *n*(B) = 4
C = {2, 4, 6, 8} ∴ *n*(C) = 4	C = {2, 4, 6} ∴ *n*(C) = 3
A∪B∪C = {1, 2, 3, 4, 5, 6, 7, 8, 9}	A∪B∪C = {1, 2, 3, 4, 5, 6, 7, 9}
∴ *n*(A∪B∪C) = 9	∴ *n*(A∪B∪C) = 8
U = {1, 2, 3, 4, 5, 6, 7, 8, 9} ∴ *n*(U) = 9	U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} ∴ *n*(U) = 10
From the above table, what is the relation between n(A∪B∪C) and n(U) in the figure no.1 and figure no. 2? And why?	

Activity 3

In a survey of students of a classroom, 40 students like orange, 35 like mango and 50 like banana. Along them, 15 like orange and mango, 20 like mango and banana, 25 like orange and banana, 5 like all the three fruits and 30 does not like either of the fruits. How shall the number of participants of the survey be found by using the Venn-diagram?

Here, O, M and B denote the set of students who like orange, mango and banana respectively.



At first, let us insert the number of students who likes all the three fruits $n(O \cap M \cap B) = 5$ and the number of students who does not like either of the fruits $n(\overline{O \cup M \cup B}) = 30$

After that, insert the number of students who likes exactly two fruits,

The number of students who likes orange and mango is $n(O \cap M) = 15$. Since 5 has already come in the number of students who likes all the three, insert the number of students who likes orange and mango only $n_0(O \cap M) = 15 - 5 = 10$. Again, since 5 has already come in the number of students who likes all the three, insert the number of students who likes mango and banana only, $n_0(M \cap B) = 20 - 5 = 15$ and the number of students who likes orange and banana only $n_0(O \cap B) = 25 - 5 = 20$.

Similarly,

40 students like orange but 5 like all the three fruits; orange, mango and banana. Similarly, 10 like orange and mango only as well as 20 like orange and banana only. Thus, insert the number of students who likes orange only, $n_0(O) = 40 - (5 + 10 + 20) = 5$

35 sutdents like mango but 5 like all the three fruits orange, mango and banana. Similarly, 10 likes orange and mango only as well as 15 like mango and banana only. Thus, insert the number of students who likes mango only, $n_0(M) = 35 - (5 + 10 + 15) = 5$

50 students likes banana but 5 likes all three fruits orange, mango and banana, 15 likes banana and mango only as well as 20 like orange and banana only. Thus, insert the number of students who like banana only, $n_0(B) = 50 - (5 + 15 + 20) = 10$.

Now, the total students $n(U) = 5 + 10 + 5 + 5 + 20 + 15 + 10 + 30 = 100$

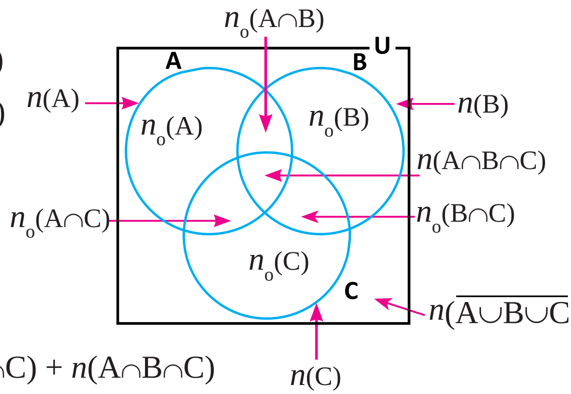
$\therefore$ The total number of participants is 100.

If A, B and C are overlapping sets, the following relations can be written from the Venn-diagram given alongside:

(a) $n_o(A\cap B) = n(A\cap B) - n(A\cap B\cap C)$

(b) $n_o(A\cap C) = n(A\cap C) - n(A\cap B\cap C)$

(c) $n_o(B\cap C) = n(B\cap C) - n(A\cap B\cap C)$



(d) $n(A) = n_o(A) + n_o(A\cap B) + n_o(A\cap C) + n(A\cap B\cap C)$

(e) $n(B) = n_o(B) + n_o(A\cap B) + n_o(B\cap C) + n(A\cap B\cap C)$

(f) $n(C) = n_o(C) + n_o(A\cap C) + n_o(B\cap C) + n(A\cap B\cap C)$

(g) $n(U) = n_o(A) + n_o(B) + n_o(C) + n_o(A\cap B) + n_o(B\cap C) + n_o(A\cap C) + n(A\cap B\cap C) + n(\overline{A\cup B\cup C})$

or, $n(U) = n(A\cup B\cup C) + n(\overline{A\cup B\cup C})$ where,

$n(A\cup B\cup C) = n_o(A) + n_o(B) + n_o(C) + n_o(A\cap B) + n_o(B\cap C) + n_o(A\cap C) + n(A\cap B\cap C)$

From the Venn-diagram with three sets, the following relations can also be written:
If A,B and C are overlapping sets, then,

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

Thus, $n(A\cup B\cup C) = n\{(A\cup B)\cup C\}$

$= n(A\cup B) + n(C) - n\{(A\cup B)\cap C\}$

$= n(A) + n(B) - n(A\cap B) + n(C) - n\{(A\cap C) \cup (B\cap C)\}$

$[\therefore (A\cup B)\cap C = (A\cup C)\cup (B\cap C)]$

$= n(A) + n(B) - n(A\cap B) + n(C) - [n(A\cap C) + n(B\cap C) - n\{(A\cap C)\cap(B\cap C)\}]$

$= n(A) + n(B) - n(A\cap B)+n(C) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$

$[\therefore (A\cup C)\cap (B\cap C)= (A\cap B\cap C)]$

$= n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$

$\therefore n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

If the sets are disjoint sets then
$n(A\cup B\cup C) = n(A) + n(B) + n(C)$

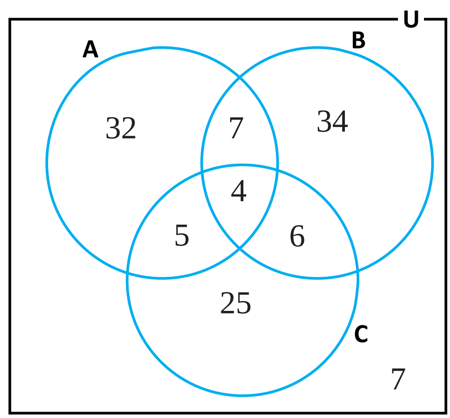
Example 1

If $n(U) = 120$, $n(A) = 48$, $n(B) = 51$, $n(C) = 40$, $n(A\cap B) = 11$, $n(B\cap C) = 10$, $n(A\cap C) = 9$, and $n(A\cap B\cap C) = 4$ then find $n(A\cup B\cup C)$ and $n(\overline{A\cup B\cup C})$. Also show the information in the Venn-diagram.

Solution

Here, given that

$n(U) = 120$, $n(A) = 48$, $n(B) = 51$, $n(C) = 40$, $n(A\cap B) = 11$, $n(B\cap C) = 10$, $n(A\cap C) = 9$, and $n(A\cap B\cap C) = 4$



We know that

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 48 + 51 + 40 - 11 - 10 - 9 + 4$

$= 113$

Again,

$n(U) = n(A\cup B\cup C) + n(\overline{A\cup B\cup C})$

or, $120 = 113 + n(\overline{A\cup B\cup C})$

or, $n(\overline{A\cup B\cup C}) = 120 - 113$

$\therefore n(\overline{A\cup B\cup C}) = 7$

The obtained information is shown in the Venn-diagram alongside.

Example 2

Among the 180 students who participated in SLC examination in 2071 from Nepal Madhyamik Vidhyaalaya, 86 passed in Science, 80 passed Maths and 76 passed in Nepali. Out of them, 26 passed in Science and Maths, 36 passed in Maths and Nepali as well as 32 passed in Science and Nepali but 20 did not pass all the subjects. Then,

a) Show the given information in the Venn-diagram.

b) Find the number of students who passed in all three subjects.

Solution

Here, n(U), n(M), n(S) and n(N) respectively denote the total number of students, the number of students who passed in Maths, the number of students who passed in Science and the number of students who passed in Nepali.

Here,

Total number of students $n(U) = 180$

The number of students who passed in Science, $n(S) = 86$

The number of students who passed in Maths, $n(M) = 80$

The number of students who passed in Nepali, $n(N) = 76$

The number of students who passed in Science and Maths, $n(S \cap M) = 26$

The number of students who passed in Maths and Nepali, $n(M \cap N) = 36$

The number of students who passed in Science and Nepali, $n(S \cap N) = 32$

The number of students who did not pass in any subject, $n(\overline{M \cup N \cup S}) = 20$

a) We know that,

$$n(U) = n(S) + n(M) + n(N) - n(S \cap M) - n(M \cap N) - n(S \cap N) + n(S \cap M \cap N) + n(\overline{S \cup M \cup N})$$

or, $180 = 86 + 80 + 76 - 26 - 36 - 32 + 20 + n(S \cap M \cap N)$

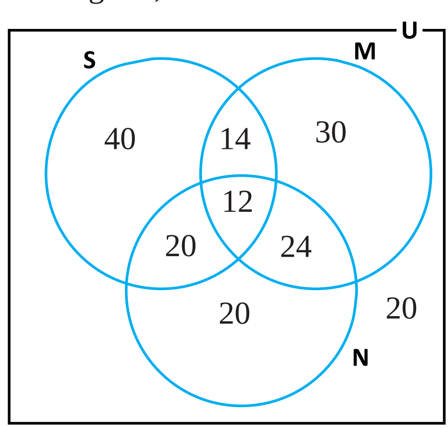
or, $180 = 168 + n(A \cap B \cap C)$

or, $n(S \cap M \cap N) = 180 - 168$

or, $n(S \cap M \cap N) = 12$

Thus, the number of students who passed in all three subjects is 12.

b) Illustrating in the Venn-diagram,



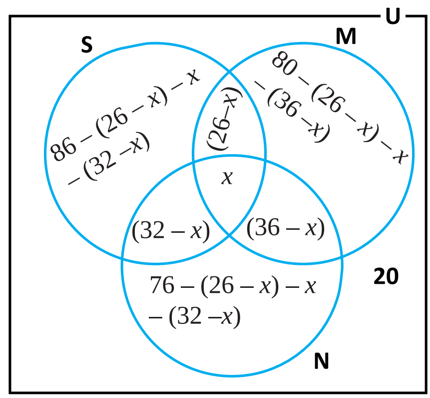
Alternative Method

Suppose the number of students who passed in all three subjects n(M∩N∩S) = x

The information is shown in the Venn-diagram.

From venn-diagram,

$\{86 - (26 - x) - x - (32 - x)\} + (26 - x) + (32 - x) + x + (36 - x) + \{80 - (26 - x) - x - (36 - x)\} + \{76 - (36 - x) - x - (32 - x)\} + 20 = 180$



or, $(28 + x) + 94 - 2x + (18 + x) + (8 + x) + 20 = 180$

or, $168 + x = 180$

or, $x = 180 - 168$

$\therefore$ x = 12

Thus the number of students who passed in all three subjects is 12.

Example 3

A school distributed medals for the students in different events of a competition. 36 got medals in dance, 12 in drama and 18 in music. If only 45 students got medals and 4 students got medals in all three events, then find the number of students who got medals in exactly two events.

Solution

Here, n(A), n(B) and n(C) denote the number of students who got medals in dance, drama and music respectively. Then, we have

The number of students who got medals in dance, n(A) = 36

The number of students who got medals in drama, n(B) = 12

The number of students who got medals in music, n(C) = 18

The number of students who got medals in at least one event, n(A∪B∪C) = 45

The number of students who got medals in all three events, n(A∩B∩C) = 4

The number of students who got medals in exactly two events

$n_o(A\cap B) + n_o(B\cap C) + n_o(A\cap C) = ?$

We know that,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

or, $45 = 36 + 12 + 18 - n(A \cap B) - n(B \cap C) - n(A \cap C) + 4$

or, $45 = 70 - n(A \cap B) - n(B \cap C) - n(A \cap C)$

or, $n(A \cap B) + n(B \cap C) + n(A \cap C) = 70 - 45$

$\therefore n(A \cap B) + n(B \cap C) + n(A \cap C) = 25$

Now,

$n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C)$

$= n(A \cap B) - n(A \cap B \cap C) + n(B \cap C) - n(A \cap B \cap C) + n(A \cap C) - n(A \cap B \cap C)$

$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 4 - 4 - 4$

$= 25 - 12$

$= 13$

Alternatively,

Let, $n_0(A \cap B) = a$, $n_0(B \cap C) = b$, $n_0(A \cap C) = C$

We know that $a + b + c = ?$

The information is shown in the Venn-diagram alongside:

From the Venn-diagram, we have:

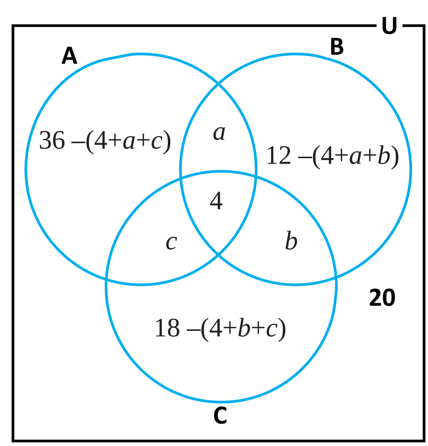
$\{36 - (4 + a + c)\} + a + 4 + b + c + \{12 - (4 + a + b)\} + \{18 - (4 + b + c))\} = 45$

or, $(32 - a - c) + 4 + a + b + c + (8 - a - b) + (14 - b - c) = 45$

or, $58 - a - b - c = 45$

or, $a + b + c = 58 - 45$

$\therefore a + b + c = 13$



Hence, the number of students who got medals in only two events is 13.

Exercise 1.2

1. In the given Venn-diagram, the elements of sets P, Q and R are illustrated. Based on this, find the values of the following sets.

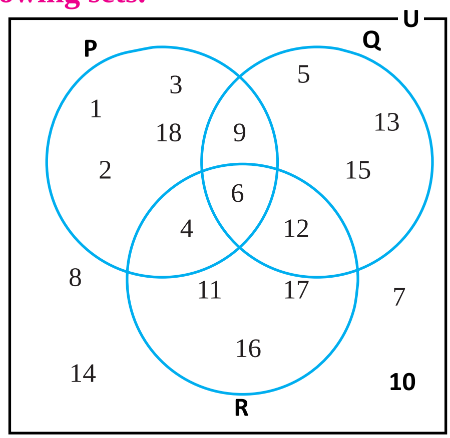
(a) $n(P)$ (b) $n(Q)$

(c) $n(P\cup Q\cup R)$ (d) $n_o(P)$

(e) $n_o(R)$ (f) $n(P\cap R)$

(g) $n(\overline{P\cup Q\cup R})$ (h) $n_o(P\cap Q)$

(i) $n(P\cap Q\cap R)$



2. If U = { positive integers less than 30},

P = {multiples of 2 less than 30},
Q = {multiples of 3 less than 30} and
R = {multiples of 5 less than 30} then

Show the relation between the sets P, Q and R in a Venn-diagram and verify the following relations:

(a) $n(P\cup Q) = n(P) + n(Q) - n(P\cap Q)$

(b) $n(P\cup Q\cup R) = n(P) + n(Q) + n(R) - n(P\cap Q) - n(Q\cap R) - n(R\cap P) + n(P\cap Q\cap R)$

(c) $n(P\cup Q\cup R) = n(P - Q) + n(Q - R) + n(R - P) + n(P\cap Q\cap R)$

3. (a) If $n(U) = 100$, $n(M) = 45$, $n(E) = 50$, $n(S) = 35$, $n(M\cap E) = 20$, $n(E\cap S) = 20$, $n(S\cap M) = 15$ and $n(M\cap E\cap S) = 5$, then find $n(\overline{M\cup E\cup S})$

(b) If $n(U) = 105$, $n(A) = 40$, $n(B) = 35$, $n(C) = 30$, $n(A\cap B) = 15$, $n(B\cap C) = 12$, $n(A\cap B\cap C) = 6$ and $n(\overline{A\cup B\cup C}) = 30$, then find $n(A\cap C)$

(c) If $n(U) = 120$, $n(M) = 50$, $n(E) = 40$, $n(S) = 45$, $n(M\cap E) = 15$, $n(E\cap S) = 15$, $n(S\cap M) = 15$ and $n(\overline{M\cup E\cup S}) = 15$, then find $n(M\cap E\cap S)$

(d) If $n(A\cup B\cup C) = 105$, $n_o(A) = 25$, $n_o(B) = 25$, $n_o(C) = 15$, $n_o(A\cap B) = 15$, $n_o(A\cap C) = 10$ and $n(A\cap B\cap C) = 10$, then find $n_o(B\cap C)$

4. a) Out of 90 students who participated in an examination, 43 passed in Science, 40 in Mathematics and 38 in Nepali. Among them, 13 passed in Science and Mathematics, 18 in Mathematics and Nepali as well as 16 passsed in Science and Nepali. Using the information, answer the following questions:

i) Show the information in the Venn-diagram.

ii) Find the number of students who did not pass in any subject.

b) In a survey of a group, 60 like tea, 45 like coffee, 30 like milk, 25 like thecoffee and tea, 20 like milk and tea, 15 like coffee and milk and 10 like all three drinks. Based on the information, answer the following questions:

i) Show the information in a Venn-diagram.

ii) Find how many people were surveyed.

c) In a survey among 60 students, 23 played volleyball, 15 played basketball and 20 played cricket. If 7 played volleyball and basketball, 5 played basketball and cricket, 4 played volleyball and cricket but 15 played neither of the games. Based on this information, answer the following questions:

i) Show the information in Venn-diagram.

ii) Find how many students played all the three games.

iii) How many played only volleyball and cricket.

5. Out of total students who participated in an examination, 40% passed in Science, 45% in Mathematics and 50% in Nepali. Similarly, 10% passed in Science and Mathematics, 20% in Mathematics and Nepali as well as 15% in Science and Nepal but 5% failed in all the three subjects. Based on the information, answer the following questions:

i) Find the percentage of students who passed in all the three subjects.

ii) find the percentage of students who passed in only one subject.

iii) Find the percentage of students who passed in only two subjects.

iv) Find the percentage of students who passed in at least one subject.

ev) Show the information in a Venn-diagram

6. The following information was obtained from the survey on a questionnaire whether they read Yubamanch or Madhupark or Muna conducted among some people in a community:

30 read Yubamanch, 25 read Madhupark, 15 read both Yubamanch and Muna, 12 read both Yubamanch and Madhupark and 9 read Madhupark only, 11 read Muna only, 5 read Yubamanch and Madhupark only but 10 read neither of the newspapers. Based on this information, answer the following questions:

a) Show the information in a Venn-diagram.

b) Find the total number of people who participated in the survey.

c) Find the number of people who read exactly two newspapers.

d) Find the number of people who read Muna.

7. In a survey among 90 people, who were asked which language film they like, 48 like Nepali, 40 like English, 31 like Hindi, 24 like Nepali and English, 19 like Hindi and English, 6 like of all the three languages and 21 did not like any. Then,

a) How many people did like both Nepali and Hindi films?

b) How many people did not like Hindi film?

b) How many people did not like both Nepali and Hindi films?

Project work

It is informed from school administration that your class has to decide the destination of educational excursion to be organized by your school. For this, form groups of all students containing 5 members each. Collect the answers from all students of different classes by asking the following questions:

Which place do you like to go for your educational excursion?

(a) Pokhara (b) Lumbini (c) Kathmandu (d) Pokhara and Lumbini (e) Lumbini and Kathmandu (f) Pokhara and Kathmandu (g) Pokhara, Lumbini and Kathmandu (h) other places than these

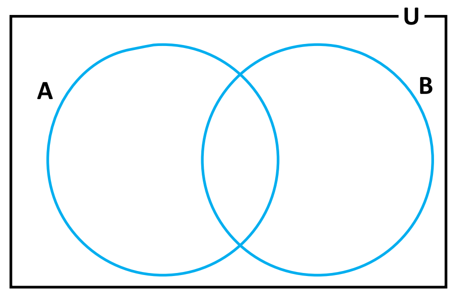
Find the number of students who like to visit only one place by illustrating the information obtained from the students answers in a Venn-diagram. Then, in turn, present the group work in the classroom.

Answers

1.	(a) 7	(b) 6	(c) 14	(d) 4	(e) 3
	(f) 2	(g) 4	(h) 1	(i) 1	
3.	(a) 20	(b) 9	(c)15	(d) 5	
4.	(a) (ii) 10	(b) (ii) 85	(c) (ii) 3	(iii) 1	
5.	(a) 5%	(b) 60%	(c) 30%,	(d) 95%	
6.	(b) 64	(c) 17	(d) 30		
7.	(a) 13	(b) 50	(c) 24		

Mixed Exercise

1. There are two overlapping sets A and B shown alongside in a Venn-diagram where $n_o(A) = 16 + x$, $n_o(B) = 5x$, $n(A \cap B) = y$ and $n(\overline{A \cup B}) = x$. Then, answer the following questions:



a) Insert the above information by drawing a Venn-diagram.

b) If n(A) = n(B), find the value of $n(\overline{A \cup B})$.

c) If n(U) = 50, find the ratio of $n(A \cap B)$ and $n(\overline{A \cup B})$.

2. A and B are the subsets of Universal set U such that $n(U) = 100$, $n(A - B) = 32 + x$, $n(B - A) = 5x$, $n(A \cap B) = x$ ₹ $n(\overline{A \cup B}) = y$.

a) Show the above information in a Venn-diagram.

b) If n(A) = n(B), find the value of $n(A \cap B)$.

c) Find the value of $n(\overline{A \cup B})$.

By what percent $n(A \cap B)$ is more or less than $(\overline{A \cup B})$? Find.

3. According to a survey of 93 women of a community, the number of women engaged in agriculture is 80 and that in sewing is 71 but the number of women engaged in other job is 10.

a) Present the information in Venn-diagram by finding the cardinality of sets.

b) Find how many women were engaged in both agriculture and sewing.

c) By how many times the number of women engaged in agriculture only is more than the number of women engaged in sewing only? Calculate it.

4. According to a survey of 1000 farmers in a community, the number of farmers cultivating potatoes was 800 and the number of farmers cultivating tomatoes was 500 but 50 crops other than these.

a) Show the information in Venn-diagram by finding the cardinality of sets.

b) Find the number of farmers who cultivate both.

c) Write the number of farmers cultivate potato only and that of tomato only in ratio.

5. In a survey of 400 people of a community, it was found that the ratio of the people having motorcycle license only and car license only was 5:3. Among them, one -fourth of the people had license of vehicles but 60 did not have any license.

a) Show the above information in a Venn-diagram.

b) From the above information, how many people had license of each vehicle?

c) Find the number of people who had license of motorcycle.

6. The information of the students of a school whether they like volleyball, football or cricket is as follows:

- 30 like volleyball and football, 20 like volleyball and cricket as well as 35 like football and cricket.
- 10 like all the three games; football, volleyball and cricket but 5 like neither of the games.

a) Represent the given information in cardinality of sets.
b) Show the information in Venn-diagram.
c) Find the total number of students in the school.
d) What percentage of students like football only?

7. The following information from a survey of 45 people of different lingual group of a community is obtained:

25 speak Nepal Bhasa, 23 speak Tamang and 15 speaks Maithili.

12 speaks Nepal Bhasa and Tamang, 5 speak Nepal Bhasa and Maithili as well as 10 speak Tamang and Maithili. 4 speak all the three languages.

Based on the information, answer the following questions:

a) Show the above information in a Venn-diagram.

b) Find how many people speak the language other than these languages; Nepal Bhasa, Tamang and Maithili.

c) How many people speak only one language? Find.

d) How many people speak both Nepal Bhasa and Tamang but do not speak the Maithili language?

Answars

1. (b) 4 (c) 3:2
2. (b)) 8 (c) 12 (d) more than 50%
3. (b) 68 (c) 4
4. (b) 350 (c) 3:1
5. (b) 250 and 190 (c) 250
6. (c) 230 (d) 19.57%
7. (b) 5 (c) 21 (d) 8