Lesson 12

Circle

12.0 Review

Experimentally verify the given statements.

(a) The perpendicular drawn from the centre of a circle bisects the chord.

(b) The line joining the centre of a circle to the midpoint of the chord is perpendicular to the chord.

(c) Chords which are equidistant from the centre of a circle are equal.

12.1 Central Angle and Inscribed Angle

Activity 1

Observe the given circles. O is the centre of the given circles. Based on this, discuss the questions below.

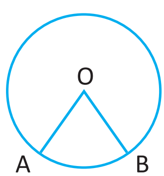


Figure (a)

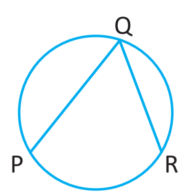


Figure (b)

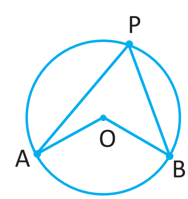


Figure (c)

(a) Where is the ∠AOB formed? What is it called?

(b) Where is the ∠PQR formed? What is it called?

(c) What is the difference between ∠AOB and ∠PQR? Compare it.

(d) In figure (c), what are arc APB and AB called?

1. The angle formed by two radii at the centre is called the central angle. In the given figure, ∠AOB is called the central angle.
2. The angle formed by joining two chords of a circle at the circumference is called the circumference (inscribed) angle. In the figure, ∠PQR is called the circumference angle.
3. If an arc is smaller than a semicircle, it is called a minor arc and if it is larger than a semicircle, it is called a major arc. Here, $\overset{\frown}{APB}$ is a major arc and $\overset{\frown}{AB}$ is a minor arc.

12.2 Relation between central angle and its corresponding arc

Activity 2

Draw circles with the centre O using a compass. Discuss the relationship between the central angles and its opposite arc.

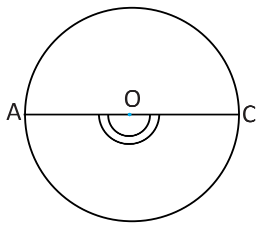


Figure 1

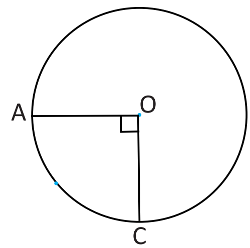


Figure 2

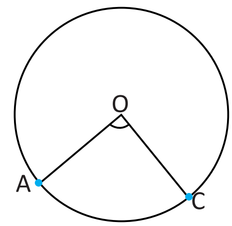


Figure 3

(a) When the central angle is 180°, discuss what part of the circumference is the arc opposite to it.

(b) When the central angle is one fourth of the circle, discuss what part of the circumference is the arc opposite to it.

(c) When the central angle is one sixth of the circle, discuss what part of the circumference is the arc opposite to it.

(d) Is there a direct relationship between the central angle and its opposite arc?

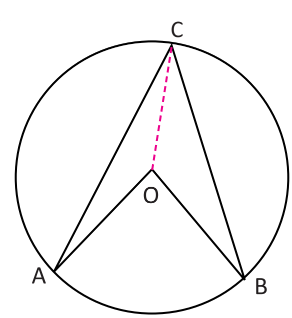
$\angle AOC \circeq \overset{\frown}{AOC}$ read as arc AOC equals in degree measure. There is direct a relation between the central angle and its opposite arc. The symbol $\circeq$ is also read as equal influence.

12.3 Relation between inscribed angle and its corresponding

Activity 3

Draw a circle with centre O by using a compass and pencil where ∠AOB is a central angle and ∠ACB is a circumference angle. O and C are joined. Are the radii of the same circle OA, OB and OC equal?

Now, what types of triangles are triangle OAC and triangle OBC? Discuss which sides and angles of these triangles are equal.



Now, in an isosceles triangle OAC,

$\angle OAC + \angle OCA + \angle AOC = 180°$

or, $2\angle OCA = 180° - \angle AOC$ .... (i) [$\angle OAC = \angle OCA$]

Again, in an isosceles triangle OBC,

$\angle OCB + \angle OBC + \angle BOC = 180°$

or, $2\angle OCB = 180° - \angle BOC$ .... (ii) [$\angle OCB = \angle OBC$]

Adding equation (i) and (ii), we get

$2(\angle OCA + \angle OCB) = 360° - (\angle AOC + \angle BOC)$

or, $2\angle ACB = 360° - \text{Reflex } \angle AOB$

or, $2\angle ACB = \angle AOB$

or, $2\angle ACB \triangleq \overset{\frown}{AB}$

The relationship between the double the angle at the circumference and its opposite arc has the equal influence. It is denoted by $2\angle ACB \triangleq \overset{\frown}{AB}$

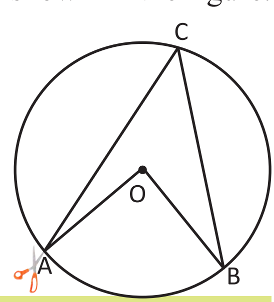
12.4 Relation between the central angle and inscribed angle

(a) Relation between the central angle and inscribed angle based on the same arc

Activity 4

(i) Using paper

Draw a central angle and inscribed angle in a chart paper as shown in the figure. Take out the central angle by cutting with a scissor. Now, fold the central angle making two equal parts and measure the inscribed angle and find the conclusion.



The central angle is double of the inscribed angle based on the same arc.

$\angle AOB = 2\angle ACB$

(ii) Experimental verification

Draw the circles having different radii as shown in the figure.

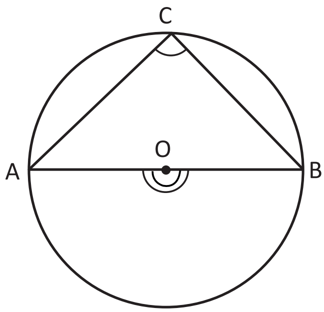


Figure 1

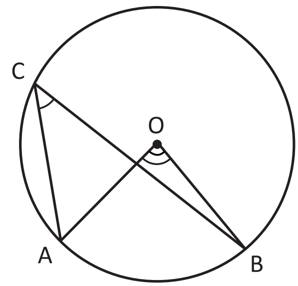


Figure 2

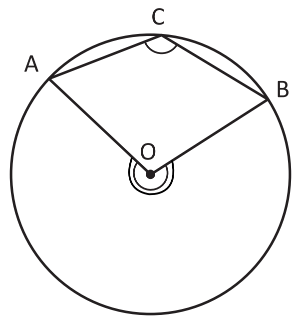


Figure 3

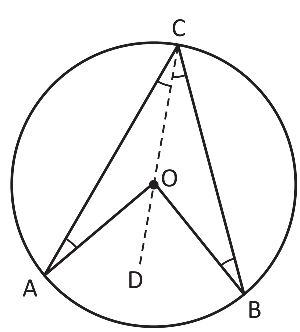
Measure the central angle AOB and the inscribed angle ACB based on the same arc AB of each circle and fill in the table below.

Figure No.	∠AOB	∠ACB	Result
1.			
2.			
3.			
Conclusion:			

(iii) Theoretical proof

Theorem 1

The central angle is double of the inscribed angle based on the same arc.



Given: O is the centre of the circle. The central angle ∠AOB and inscribed angle ∠ACB are based on the same arc AB.

To be prove: ∠AOB = 2∠ACB

Construction: Points C and O are joined and produced CO to the point D.

Proof

S.N.	Statement	Reason
1.	In ΔAOC, (i) $\angle OAC = \angle OCA$ (ii) $\angle AOD = \angle OAC + \angle OCA$ (iii) $\angle AOD = \angle OCA + \angle OCA$ $= 2\angle OCA$	(i) OA = OC (radii of the same circle), So base angles of isosleces Δ. (ii) Exterior and opposite interior angle of ΔAOC (iii) from statement (i) and (ii)
2.	In ΔBOC, $\angle BOD = 2\angle OCB$	Same as above facts and reasons.
3.	$\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$	From statements 1(iii) and (2) above.
4.	$\therefore \angle AOB = 2\angle ACB$	From statement (3), by whole partaxiom.
Conclusion: The central angle is double of the inscribed angle based on the same arc.		

Proved.

The inscribed angle is half of the centre angle based on the same arc. That is, the central angle is twice the angle on the circumference. As in the above figure, $\angle AOB = 2\angle ACB$.

Activity 5

How to show the angle in the circumference of the semicircle is a right angle by experimental verification.

(ii) Experimental verification

Draw the circles having different radii as shown in the figures. The angle$\angle ACB$ is on the circumference based on the diameter. Write the conclusion by taking the value of $\angle ACB$ in the table below.

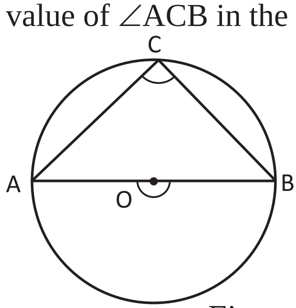


Figure 1

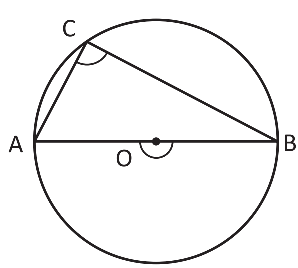


Figure 2

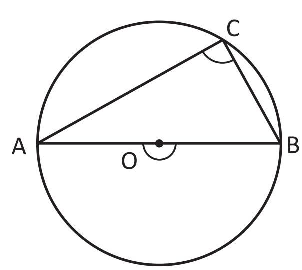
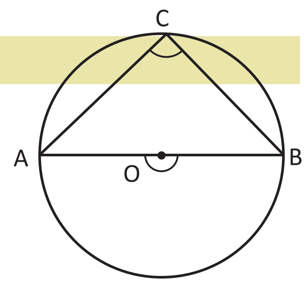


Figure 3

Measure the angle ∠ACB of the semicircle of each circle and fill in the table below.

Figure No.	∠ACB	Result
1.		
2.		
3.		
Conclusion:		

(ii) Theoretical proof



Given: O is the centre of a circle in which AOB is the diameter of the circle. ∠ACB is the angle of the circumference based on the diameter.

To Prove: ∠ACB = 90°

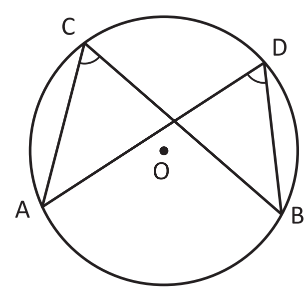
	Statement	Reason
1.	$\angle ACB = \frac{1}{2}\angle AOB$	Inscribed angle is half of the central angle standing on the same arc.
2.	∠AOB =180°	Being ∠AOB is a straight angle
3.	$\angle ACB = \frac{1}{2} \times 180° = 90°$	From statement (1) and (2) above
Conclusion: An angle in a semicircle is a right angle.		

Proved.

(b) Relation between the angles in the circumference based on the same arc

Activity 6

(i) Using paper



On a chart paper, draw two angles in the circumference of the circle as shown in the figure. With the help of scissors, cut one angle from the circumference and fold it to the other angle. Write a conclusion based on this.

Inscribed angles are equal when they base on the same arc. ∠ACB = ∠ADB

(ii) Experimental verification

Draw the circles having different radii as shown in the figure. The angle ∠ACB and ∠ADB are the angles on the circumference based on the same arc AB.

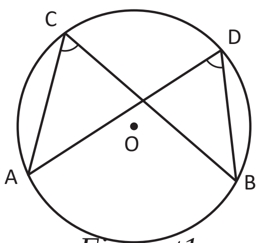


Figure t1

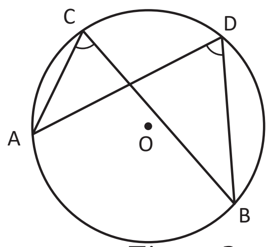


Figure 2

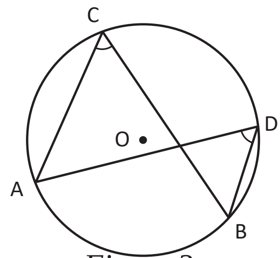


Figure 3

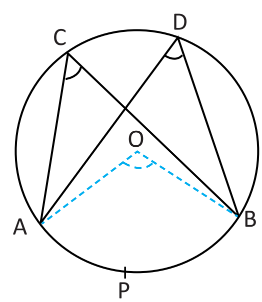
Measure the circumference angles ∠ACB and ∠ADB based on the same arc AB of each circle. Fill in the table below and write the conclusion.

Figure No.	∠ACB	∠ADB	Result
1.			
2.			
3.			
Conclusion:			

(iii) Theoretical proof

Theorem 2

The angles on the circumference of a circle, based on the same arc are equal.



Given: O is the centre of a circle in which inscribed angles ∠ACB and ∠ADB are based on the same arc AB.

To prove: ∠ACB = ∠ADB

Construction: Join centre O of the circle with the points A and B successively.

Proof

	Statements	Reasons
1.	∠AOB = 2∠ACB	The circumference angle and the central angle are based on the same arc APB.
2.	∠AOB = 2∠ADB	The circumference angle and the central angle are based on the same arc APB.
3.	2∠ACB = 2∠ADB or, ∠ACB = ∠ADB	From statement (1) and (2)
Conclusion: The angles on the circumference of a circle, based on the same arc are equal.		

Proved.

12.5 The Relation between opposite angles of cyclic quadrilateral

(i) Experimental verification

Draw circles having different radii as shown in the figure. Draw the cyclic quadrilateral ABCD in each circle.

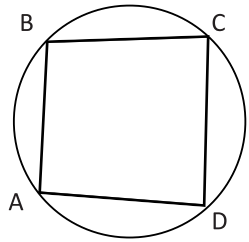


Figure 1

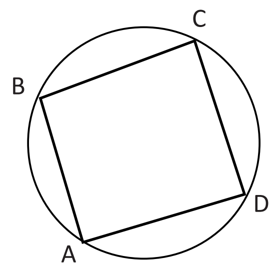


Figure 2

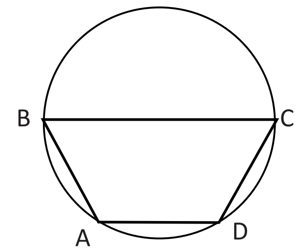


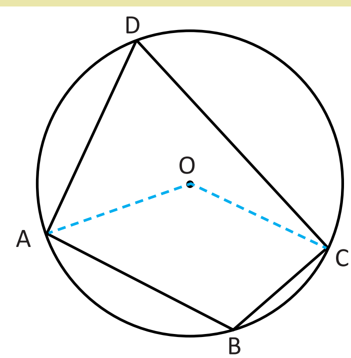
Figure 3

Measure the circumference angles and fill in the table below and write the conclsion.

Figure No.	∠DAB	∠ABC	∠BCD	∠ADC	∠DAB + ∠BCD	∠ABC + ∠ADC	Result
1.							
2.							
3.							
Conclusion:							

(ii) Theoretical proof

Theorem 3



The opposite angles of a cyclic quadrilateral are supplementary.

Given: O is the centre of a circle. ABCD is a cyclic quadrilateral.

To prove: $\angle ABC + \angle ADC = 180^{\circ}$

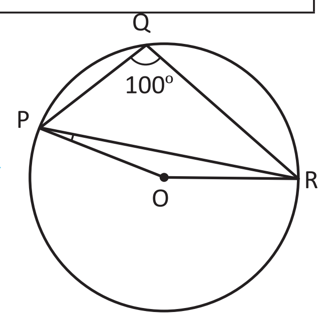
$\angle BCD + \angle BAD = 180^{\circ}$

Construction: Join centre O of the circle with the points A and C successively.

Proof

	Statements	Reasons
1.	Obtuse $\angle AOC = 2\angle ADC$	The central angle and the circumference angle based on the same arc APB.
2.	Reflex $\angle AOC = 2\angle ABC$	The central angle and the circumference angle based on the same arc ADC.
3.	$2\angle ADC + 2\angle ABC =$ Obtuse $\angle AOC +$ Reflex $\angle AOC$ or, $2(\angle ADC + \angle ABC) = 360°$ or, $\angle ADC + \angle ABC = \frac{360°}{2} = 180°$ $\therefore \angle ADC + \angle ABC = 180°$	From statement (1) and (2), the sum of the angles around the point O is 360°
4.	Similarly, $\angle DAB + \angle DCB = 180°$	Similar as above.

Example 1



In the given figure, $\angle PQR = 100°$ and the points P, Q and R are the circumference points of the circle with the centre O. What is the value of $\angle OPR$< Find.

Solution

According to the figure,

(i) Reflex angle POR $= 2 \times \angle PQR = 2 \times 100° = 200°$

[$\because$ The central angle and the inscribed angle based on the same arc PR]

(ii) Reflex angle POR + obtuse angle POR = 360° [$\because$ Sum of the angles around the point O]

$200° +$ obtuse $\angle POR = 360°$

$\angle POR = 360° – 200° = 160°$

(iii) Again, ΔPOR is an isosceles triangle. So that, $\angle OPR = \angle ORP$

$\angle OPR + \angle ORP + \angle POR = 180°$ [$\because$ The sum of the angles of a triangle]

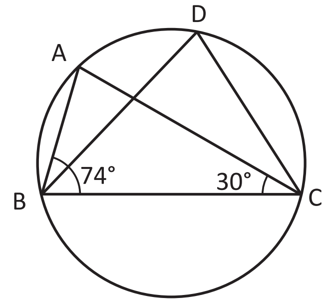
or, $\angle OPR + \angle OPR + 160° = 180°$ [$\because$ $\angle OPR = \angle OPR$]

or, $2\angle OPR = 180° – 160° = 20°$

or, $\angle OPR = \frac{20°}{2} = 10°$

Example 2

In the adjoining figure, ∠ABC = 74° and ∠ACB = 30°, then find the value of angle ∠BDC.



Solutions

Here,

(i) In the triangle ABC, ∠ABC + ∠ACB + ∠BAC =180°

[∵ Sum of interior angles of triangle.]

or, 74° + 30° +∠BAC = 180°

or, 104° +∠BAC = 180°

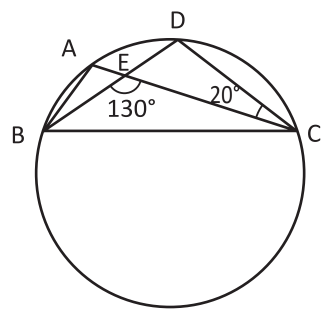
or, ∠BAC = 180° – 104° = 76°

∴ ∠BDC = 76°

(ii) BDC = BAC [∵ Inscribed angles stending on the same arc]

Example 3

In the adjoining figure, A, B, C and D are four points on the circumference of the circle. The chords AC and BD are intersecting at a point E. If ∠BEC = 130° and ∠ECD = 20°, find the value of ∠BAC.



Solution

(i) ∠BEC = 130° and ∠ECD = 20°

∠BEC + ∠CED = 180° [∵ The sum of the angles in a straight line]

or, ∠CED = 180° – ∠BEC = 180° – 130° = 50°

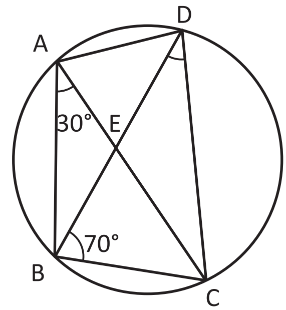
(ii) Again, ∠EDC + ∠CED + ∠ECD = 180° [∵ The sum of the angles of a triangle]

or, ∠EDC = 180° – 50° – 20°= 110°

(iii) or, ∠EDC = ∠BAC [∵ Inscribed angles standing on the same arc BC]

or, ∠BAC = 110°

Example 4



In a cyclic quadrilateral ABCD, the diagonals AC and BD are intersecting at a point E. If ∠DBC = 70° and ∠BAC = 30°, then find the value of ∠BCD. Also, if AB = BC, what is the value of ∠ECD?

Solution

In a cyclic quadrilateral ABCD, the diagonals AC and BD are intersecting at a point E.

(i) ∠DAC = ∠DBC = 70° [Inscribed angles standing on the same arc CD]

(ii) ∠DAB = ∠DAC + ∠BAC = 70° + 30° = 100° [By whole part axiom]

(iii) Again ∠BCD + ∠DAB = 180° [The sum of opposite angles of a cycle quadrilateral]

or, ∠BCD + 100° = 180°

or, ∠BCD + 100° = 180° – 100° = 80°

(iv) Again ∠BAC = ∠ACB = 30° [Being AB = BC]

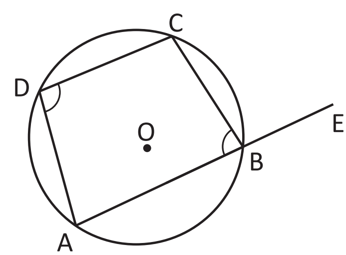
∠BCD = ∠BCA + ∠ACD = 80°

or, 30° + ∠ACD = 80°

or, ∠ACD = 80° – 30° = 50°

∴ ∠AXΔ = ∠ECD = 50°

Example 5



In the given figure alongside, ABCD is cyclic quadrilateral. If the side AB is produced to the point E then prove that ∠ADC = ∠CBE.

Solution

Given: ABCD is a cyclic quadrilateral of a circle with centre O. The side of cyclic quadrilateral AB is produced to the point E

To Prove: ∠ADC = ∠CBE

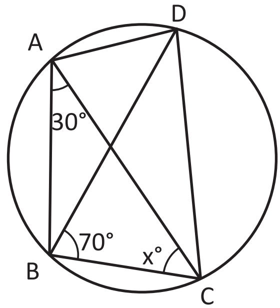
Proof:

	Statements	Reasons
1.	∠ADC + ∠ABC = 180°	The sum of opposite angles of a cyclic quadrilateral
2.	∠ABC + ∠CBE = 180°	Being straight angle
3.	∠ADC + ∠ABC = ∠ABC + ∠CBE or, ∠ADC = ∠CBE	From statement (1) and (2) above.

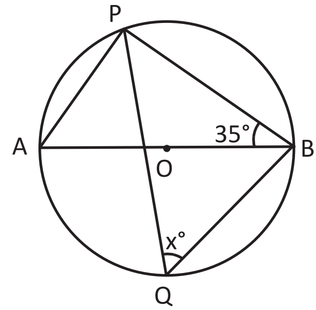
Proved.

Exercise 12

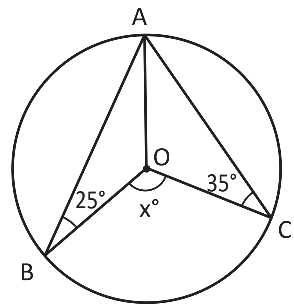
1. If O is the centre of the following circles, find the value of *x*.



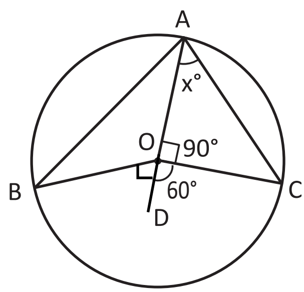
(a)



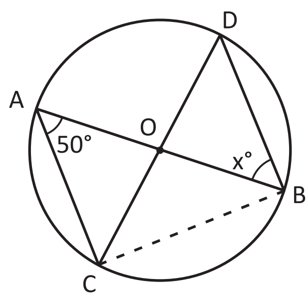
(b)



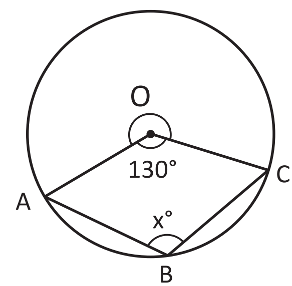
(c)



(d)

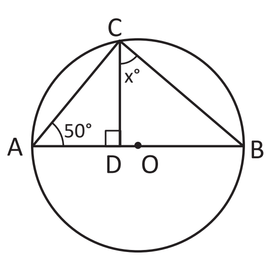


(e)

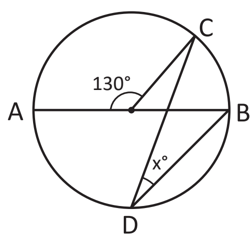


(f)

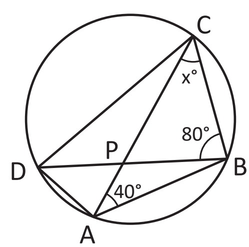
2. In the following figure, find the value of *x*.



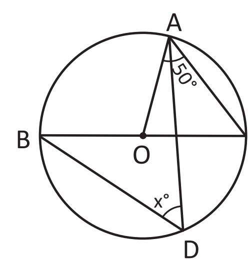
(a)



(b)

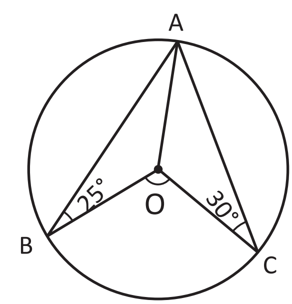


(c)

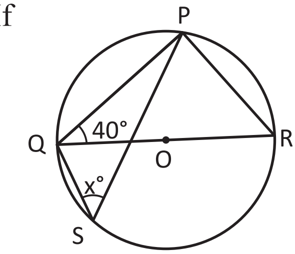


(d)

3. (a) In the adjoining figure, O is the centre of circle, ∠OBA = 25° and ∠OCA = 30° find the value of obture ∠BOC.



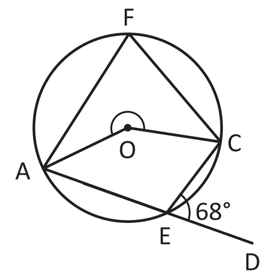
(b) In the adjoining figure, O is the centre of the circle. If $\angle PQR = 40°$ and $\angle PSQ = x°$, find the value of x.



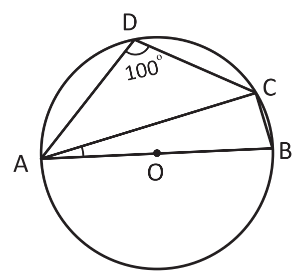
(c) In the given figure, O is the centre of the circle. FAEC is a cyclic quadrilateral. If $\angle CED = 68°$, then

(i) Find the value of $\angle AFC$.

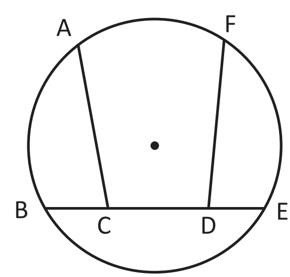
(ii) Find the reflex $\angle AOC$.



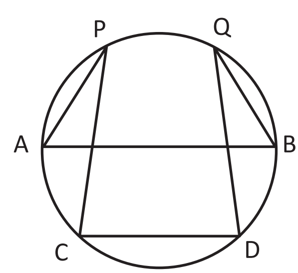
(d) In the given figure, AOB is a diameter of the circle. If $\angle ADC = 100°$, then find the value of $\angle BAC$.



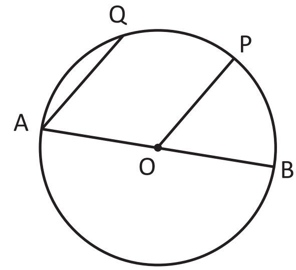
4. (a) In the adjoining figure, BC = DE and $\overset{\frown}{AB} = \overset{\frown}{FE}$ Prove that $\angle ACB = \angle FDE$.



(b) In the given figure, if $\angle APC = \angle BQD$, then prove that AB//CD.



(c) In the adjoining figure, O is the centre of the circle. If arc PQ = arc PB, then prove that AQ// OP.

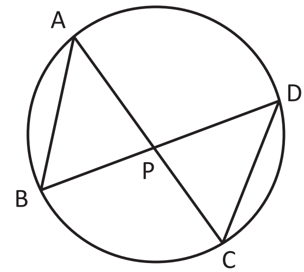


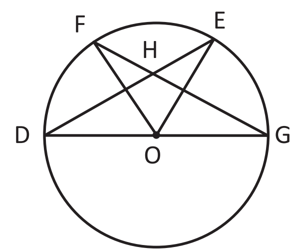
(d) In the given figure, chords AC and BD are intersected at a point P. If PB = PC then prove that:

(i) Chord AB = chord DC.

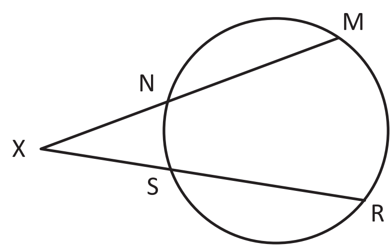
(ii) Chord AC = chord BD.

(ii) Arc ABC = arc BCD.

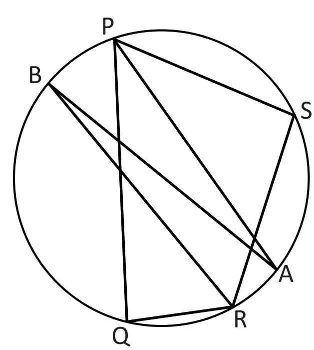




5. In the figure, O is the centre of the circle. If the chords DE and FG are intersected at a point H, prove that: $\angle DOF + \angle EOG = 2\angle EHG$.

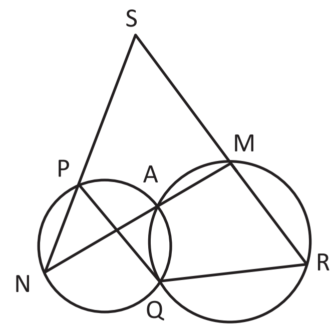


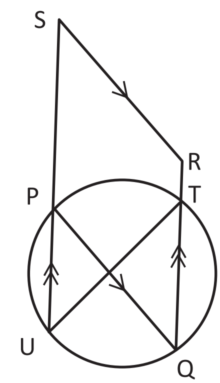
6. In the figure, chords MN and RS of the circle intersect externally at the point X. Prove that: $\angle MXR \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{MR} - \overset{\frown}{NS})$.



7. PQRS is a cyclic quadrilateral. If the bisectors of the $\angle QPS$ and $\angle QRS$ meet the circle at A and B respectively, prove that AB is a diameter of the circle.

8. In the given figure, NPS, MAN and RMS are straight lines. Prove that PQRS is a cyclic quadrilateral.





9. In the given figure, PQRS is a parallelogram. Prove that UTRS is a cyclic quadrilateral.

Practical work and project

1. Make models of paper to show the relationship between the central angle and the inscribed angle, and the arcs and chords related to them. Present them in the classroom.
2. Draw three pairs of equal circles ABP and CDQ having centres X and Y respectively. Join chords AB and CD making equal arcs AB and CD. Measure AB and CD and enter the result in a table.
   - Does chord AB = chord CD?
   - Are the angles subtended by chords AB and CD at the centre equal?
   - Is the angle subtended by the chord AB at the circumference of a circle half of the central angle? Fine thread or wire and tracing paper can be used for this work.

Answers

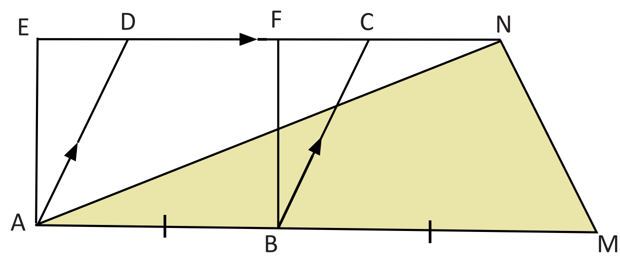
1. (a) 80° (b) 55° (c) 120° (d) 75° (e) 50° (f) 115°
2. (a) 50° (b) 25° (c) 60° (d) 50°
3. (a) 110° (b) 50° (c) (i) 68° (ii) 136° (d) 10°

Show the answers from 4 to 9 to the teachers.

Mixed Exercise

1. **In the given figure, ABCD is a parallelogram and ABFE is a rectangle.**

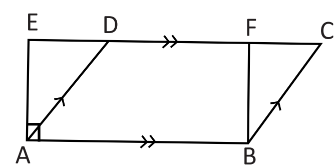
   (a) What is the relationship between the area of the parallelogram and rectangle? Write.

   (b) In the figure, if AB = BM, then write the relationship between the parallelogram ABCD and triangle AMN.

2. **A parallelogram ABCD and a rectangle ABFE are on the same base AB and between the same parallel lines AB and EC.**

   (a) What is the relationship between the area of the parallelogram ABCD and rectangle ABFE? Write it.

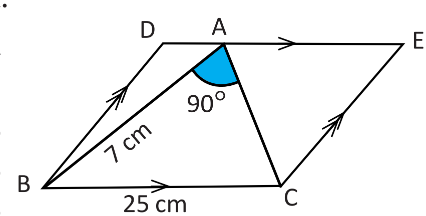
   (b) If the area of the rectangle ABFE is 35 cm^2, what is the area of the parallelogram ABCD? Find.

   (c) Construct a parallelogram ABCD having the side AB = 7 cm, BC = 5 cm and ∠ABC = 120°. Construct a rectangle ABFE whose area is equal to the area of that parallelogram.

   (d) Are the triangle AED and triangle BFC congruent? Write with reason.

3. **A parallelogram BCED and a triangle ABC are on the same base BC and between the same parallel lines BC and DE, where ∠BAC = 90°, AB = 7 cm and BC = 25 cm.**

   (a) What is the measurement of AC? Find.

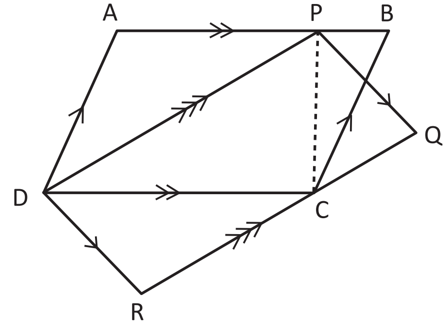
   

   (b) What is the area of parallelogram BCED? Find.

   (c) Theoretically prove that the relationship between the area of the parallelogram ABCD and triangle ABC.

   (d) Construct a triangle ABC, where AC = 5 cm, AB = 4 cm and ∠BAC = 45°. Also construct a parallelogram ADMN whose area is equal to the area of the triangle.

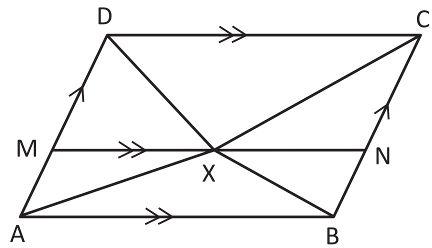
4. In the given figure, ABCD and PQRD are two parallelograms.



(a) Find the relation between the parallelograms ABCD and PQRD.

(b) If the base and height of the parallelogram ABCD are 8 cm and 7 cm respectively, find the area of the parallelogram PQRD.

5. In the given figure, ABCD is a parallelogram. X is a point inside it. If MN//AB then,



(a) Prove that the sum of the area of triangles XCD and XAB is equal to the half of the area of the parallelogram ABCD.

(b) What is the relationship among the triangle AXD, triangle BCX and parallelogram ABCD?

6. (a) Construct a parallelogram ABCD having AB = 5 cm, BC = 4 cm and $\angle ABC = 60°$. Also construct a triangle PBE equal in area to the parallelogram having a side PB = 5.6 cm.

(b) According to question (a) find the height of the parallelogram ABCD and then find the area of triangle PBE.

7. (a) Construct a parallelogram ABCD having AB = 7 cm, BC = 5 cm and $\angle ABC = 120°$. Also, construct the rectangle ABFE equal in area to the parallelogram.

(b) Find the side BF of the rectangle ABFE formed according to question no. (a) and also find the area of the parallelogram ABCD.

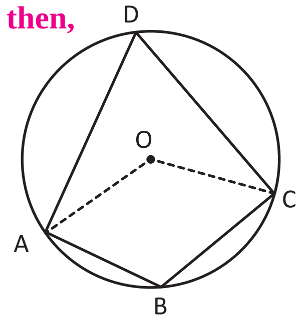
8. If a circle with centre O has a central angle $\angle BOC$ and inscribed angle $\angle BDC$ based on the same arc BC, answer the following questions.

(a) Write the relation between $\angle BOC$ and $\angle BDC$.

(b) Experimentally verify that the relationship between the central angle $\angle BOC$ and inscribed angle $\angle BDC$.

(c) The measurement of the central angle is ($7x°$) and the inscribed angle is $(3x + 5)°$, then find the value of x.

9. In the given figure, ABCD is a cyclic quadrilateral then,

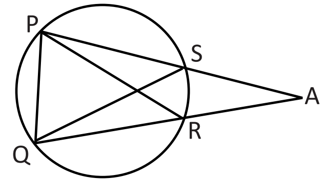


(a) Write the relation of $\angle ABC$ and $\angle ADC$.

(b) Prove that $\angle ADC = \frac{1}{2} \angle AOC$.

(c) If $\angle ABC = 120°$, what is the value of $\angle AOC$?

10. O is the centre of the given circle and P, Q, R and S are the circumference points on it. If AP = AQ then,

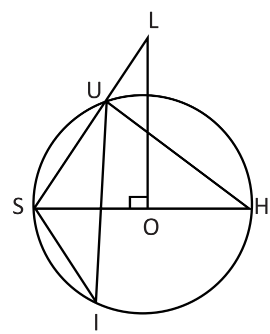


(a) Write the name of the inscribed angles based on the arc PQ.

(b) If $\angle PSQ = 60°$ then what is the measurement of the angle $\angle PRQ$?

(c) Prove that: PR = QS.

11. In the given figure, O is the centre of the circle and SH is a diameter. S, I, H and U are the circumference points and L is an external point HereLO is perpendicular to SH.



(a) What is the measurement of angle $\angle SUH$? Write with reason.

(b) Prove that: $\angle SIU = \angle OLS$.

(c) If $\angle USH = 50°$, then what is the measurement of $\angle SIU$? Find.

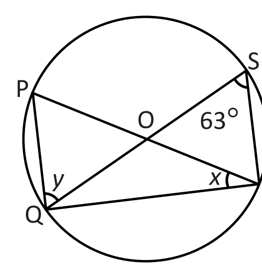
12. In the given figure, O is the centre of circle, $\angle PQS$ = y, $\angle QSR = 63°$ and $\angle PRQ = x$.

(a) What is the measurement of angle $\angle PQR$? Write with reason.

(b) What is the value of angle $\angle POS$? Find.

(c) Prove that: x + y = 90°.

(d) Prove that: ΔQOR is an isosceles triangle.



Answers

Show to your teacher.