

13.0 Review

The marks obtained in Mathematics by 27 students of class 10 in the second terminal examination are given below. On the basis of this, answer the following questions.

25, 15, 30, 22, 27, 12, 25, 30, 22, 24, 15, 23, 19, 27, 28, 17, 19, 22, 25, 15, 14, 13, 28, 26, 18, 20, 22

- What is the average marks obtained in Mathematics?
- How many students have scored less and more than the average marks?
- Find the median, the first quartile and the third quartile on the basis of the data given above.
- What is the same mark obtained by maximum number of students? How many have got it? What is it called?

13.1 Mean

Activity 1

In a survey conducted among the students of class 10 of a community school about how much rupees they bring each day for their tiffin. The following results were obtained.

3 bring Rs. 25	6 bring Rs. 30	7 bring Rs. 35
4 bring Rs. 40	4 bring Rs. 45	1 bring Rs. 50

- Present the above data in table
- Find one day's average expenditure of the grade 10 students.

To find the average value (mean) from the data given above, the following formula is applied.

$$\begin{aligned}
 \text{Mean } \bar{X} &= \frac{(f_1 x_1 + f_2 x_2 + \dots + f_n x_n)}{f_1 + f_2 + \dots + f_n} \\
 &= \frac{\sum fx}{\sum f} \\
 &= \frac{\sum fx}{n}
 \end{aligned}$$

Activity 2

If the expansion of the raw data is too much, then its table will be so large. In that case, how to find the median?



If we have the data as you said, the mean can be calculated by making continuous series.



For example, the obtained marks of 40 students of class 9 of a school in Mathematics are as follows.

25, 10, 31, 22, 37, 42, 45, 37, 32, 34, 45, 40, 29, 27, 28, 17, 19, 22, 25, 33

15, 14, 13, 28, 36, 38, 41, 42, 39, 25, 24, 31, 21, 22, 25, 26, 35, 36, 39, 49

The median can be calculated by placing the above data in a continuous series.

- (a) The lowest obtained mark is 10 and the highest obtained mark is 49. We can make a table with the class interval of 10 as given below.

Description of the marks obtained by class 9 students in Mathematics

Marks obtained (X)	Tally bars	No. of students (f)
10 - 20		6
20 - 30		14
30 - 40		13
40 - 50		7
Total number of students		40

To find the mean of the continuous series first of all we have to find the mid value of each class interval.

$$\text{Mid-value (m)} = \frac{\text{Lower limit of a class interval} + \text{Upper limit of a class interval}}{2}$$

After that, as in the discrete series, mean is by placing 'm' in the place of x. calculated.

Description of the marks obtained by class 9 students in mid-value.

Marks obtained (X)	No. of students (f)	Mid value (m)	fm
10-20	6	$\frac{10+20}{2} = 15$	90
20-30	14	$\frac{20+30}{2} = 25$	350
30-40	13	$\frac{30+40}{2} = 35$	455
40-50	7	$\frac{40+50}{2} = 45$	315
	N = 40		$\Sigma fm = 1210$

$$\bar{X} = \frac{\Sigma fxm}{N} = \frac{1210}{40} = 30.25$$

The above table is called frequency table. The number of students of each class interval is called frequency of that class interval. This method to find the mean is called direct method.

Alternative method

Description of the marks obtained by class 9 students in Mathematics

Marks obtained X	No. of students f	Mid-value m	$d = m - A$	fd
10 – 20	6	$\frac{10+20}{2} = 15$	$15 - 25 = -10$	- 60
20 – 30	14	$\frac{20+30}{2} = 25$	$25 - 25 = 0$	0
30 – 40	13	$\frac{30+40}{2} = 35$	$35 - 25 = 10$	130
40 – 50	7	$\frac{40+50}{2} = 45$	$45 - 25 = 20$	140
	N = 40			$\Sigma fd = 210$

Let, assumed mean (A) = 25.

The deviation between mid-value and assumed mean = d

$$\begin{aligned}
 \text{Mean } \bar{X} &= A + \frac{\Sigma fd}{N} \\
 &= 25 + \frac{210}{40} \\
 &= 25 + 5.25 = 30.25
 \end{aligned}$$

The actual mean of the data can also be calculated by considering the assumed mean. The actual mean is calculated by considering any mid-value and any number as assumed mean. For this, the deviation between mid-value and assumed mean (d) should be found.

Example 1

If $\sum fm = 2700$ and $N = 50$, then find mean \bar{X} .

Solution

Here, $\sum fm = 2700$

$N = 50$

Mean $\bar{X} = ?$

$$\begin{aligned}\text{We know that } \bar{X} &= \frac{\sum fm}{N} \\ &= \frac{2700}{50} \\ &= 54\end{aligned}$$

Hence, mean (\bar{X}) = 54

Example 2

The weight of 100 students of Ganesh Secondary School is given in the table below. Find the mean weight of the students from this data.

Weight in (Kg)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	18	27	20	17	6

Solution

Here, the details of the weight of students

Weight in (Kg) (X)	No. of students (f)	Mid value (m)	fm
20 – 30	27	$\frac{20 + 30}{2} = 25$	675
30 – 40	20	$\frac{30 + 40}{2} = 35$	700
40 – 50	17	$\frac{40 + 50}{2} = 45$	765
50 – 60	6	$\frac{50 + 60}{2} = 55$	330
	$\Sigma f = N = 88$		$\Sigma fm = 2740$

We know that, mean $\bar{X} = \frac{\Sigma fm}{N} = \frac{2740}{88} = 31.14$

Hence, the mean weight of students $(\bar{X}) = 31.14$

Alternative method

Here, let the assumed mean (A) = 35

Description of the weight of students

Marks obtained (X)	No. of students (f)	Mid value (m)	$d = m - 35$	fd
10 – 20	18	15	-20	-360
20 – 30	27	25	-10	-270
30 – 40	20	35	0	0
40 – 50	17	45	10	170
50 – 60	6	55	20	120
	$N = 88$			$\Sigma fd = -340$

We know that, mean $(\bar{X}) = A + \frac{\Sigma fd}{N}$
 $= 35 + \frac{-340}{88}$
 $= 35 - 3.86 = 31.14$

Hence, the mean weight of the students $(\bar{X}) = 31.14$

Example 3

The assumed mean of any data (A) = 40, $\sum fd = 20$ and $N = 10$, then find the mean \bar{X} .

Solution

Here, $A = 40$,

$\sum fd = 20$ and

$N = 10$

mean (\bar{X}) = ?

$$\begin{aligned}\text{We know that, mean } (\bar{X}) &= A + \frac{\sum fd}{N} \\ &= 40 + \frac{20}{10} \\ &= 40 + 2 \\ &= 42\end{aligned}$$

Hence, the mean (\bar{X}) = 42

Example 4

The data prepared on the basis of the weight of the people of Janajagriti Tole are given in the table. Find the average weight (mean) based on the table.

Weight (Kg)	0-10	10-20	20-30	30-40	40-50	50-60
No. of people	10	18	25	20	12	5

Solution

Here, let assumed mean (A) = 25 and length of class interval (h) = 10

Description of the weight of people of Janajagriti Tole

Weight in Kg. (X)	No. of people (f)	Mid value (m)	$d = m - 25$	fd
0-10	10	5	-20	-200
10-20	18	15	-10	-180
20-30	25	25	0	0
30-40	20	35	10	200
40-50	12	45	20	240
50-60	5	55	30	150
	$\sum f = N = 90$			$\sum fd = 210$

We know that, mean $(\bar{X}) = A + \frac{\sum fd}{N}$

$$= 25 + \frac{210}{90}$$

$$= 25 + 2.33 = 27.33$$

Hence, the mean weight of the people $(\bar{X}) = 27.33$

Example 5

The data prepared based on the height of students of class 11 and 12 of Shanti Secondary School is given in the table. If the average height of the students $\bar{X} = 157.75$ cm, find the value of p.

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No. of students	2	5	8	p	7	5	9

Solution

Here, average height of the students $(\bar{X}) = 157.75$

Description of the height of the students of class 11 and 12 of Shanti

Secondary School

Height (cm) (X)	No. of students (f)	Mid-value (m)	fm
140-145	2	142.5	285
145-150	5	147.5	737.5
150-155	8	152.5	1220
155-160	p	157.5	157.5p
160-165	7	162.5	1137.5
165-170	5	167.5	837.5
170-175	3	172.5	517.5
	$\sum f = N = 30 + p$		$\sum fm = 4735 + 157.5 p$

We know that, mean $(\bar{X}) = \frac{\sum fm}{N}$

$$157.75 = \frac{4735 + 157.5p}{30 + p}$$

$$\text{or, } 4732.5 + 157.75p = 4735 + 157.5p$$

$$\text{or, } 157.75p - 157.50p = 4735 - 4732.5$$

$$\text{or, } 0.25p = 2.5$$

$$\text{or, } p = 10$$

Hence, the required value of $p = 10$.

Example 6

The number of people entering into the Balaju Park from 7 am to 8 am according to their age is given below. Construct a frequency table of class interval of 10 from the given data and find the average age of people visiting the park.

7, 22, 32, 47, 59, 16, 36, 17, 23, 39, 49, 31, 21, 24, 41, 12, 49, 21, 9, 8, 51, 36, 35, 18.

Solution

Here, tabulating the given data in the frequency distribution table

Height (cm) (X)	Tally Bars	frequency (f)	Mid-value (m)	fm
0-10		3	5	15
10-20		4	15	60
20-30		5	25	125
30-40		6	35	210
40-50		4	45	180
50-60		2	55	110
		$\sum f = N = 24$		$\sum fm = 700$

We know that, mean $(\bar{X}) = \frac{\sum fm}{N}$

$$= \frac{700}{24}$$

$$= 29.17$$

$$(\bar{X}) = 29.17$$

Hence, the average age of people $(\bar{X}) = 29.17$ years.

Exercise 13.1**1. Find the mean in the following condition:**

(i) 35, 36, 42, 45, 48, 52, 58, 60

(ii) 13.5, 14.2, 15.8, 15.2, 16.9, 16.5, 17.4, 19.3, 15.3, 15.9

(iii)

X	5	8	10	12	14	16
f	4	5	8	10	2	2

(iv) Details of the goals scored by players in the national football league.

Goals	12	13	14	15	16	17
Number of students	2	4	6	12	10	6

2. Find the mean of the following data by direct method and short-cut method.

(a) The details age of passengers travelling in a bus

Age (year)	0-10	10-20	20-30	30-40	40-50
No. of persons	5	9	15	7	4

(b) The details of the marks obtained by class 10 students in science.

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	1	4	10	8	7	5

(c) Details of daily wages of workers

Wages (Rs)	200-400	400-600	600-800	800-1000	1000-1200
No. of workers (Rs.)	3	7	10	6	4

(d) The details of the marks obtained by class 10 students in Mathematics.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	7	5	6	12	8	2

3. Find the unknown value of the following data.(a) $\bar{X} = 49$, $\sum fm = 980$, $N = ?$ (b) $\bar{X} = 102.25$, $N = 8$, $\sum fm = ?$ (c) $A = 100$, $\bar{x} = 90$, $\sum fd = ?$, $N = 10$ (d) $\bar{X} = 41.75$, $\sum fd = 270$, $N = 40$, $A = ?$

4. (a) In the given condition, if the mean (\bar{X}) is 32.5, find the value of k.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	10	k	35	15	10

- (b) In the given condition, if the mean (\bar{X}) is 46.2, find the value of p.

X	0-20	20-40	40-60	60-80	80-100
f	35	400	350	p	65

- (c) In the given condition, if the mean (\bar{X}) is 36.4, find the value of y.

Marks obtained	16-24	24-32	32-40	40-48	48-56	56-64
No. of students	6	8	y	8	4	2

- (d) In the given condition, the mean of daily expenditure (\bar{X}) is 264.67, find the value of the unknown frequency.

Daily expenditure	0-100	100-200	200-300	300-400	400-500	500-600
No. of students	20	30	?	20	18	12

5. Prepare a frequency distribution table from the given raw data and find the mean (\bar{X}).

- (a) 15, 51, 32, 12, 32, 33, 23, 43, 35, 46, 57, 19, 59, 25, 20, 38, 16, 45, 39, 40 (Class interval 10)
- (b) 25, 15, 24, 42, 22, 35, 34, 41, 33, 38, 54, 50, 36, 40, 27, 18, 35, 16, 51, 31, 23, 9, 16, 23, 31, 51, 7, 30, 17, 40, 60, 32, 50, 10, 23, 12, 21, 28, 37, 20, 58, 39, 10, 41, 13 (Class interval 5)

6. (a) Find the mean of the following data.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	14	10	8	10

- (b)
- | | | | | | | |
|-------------------|------|--------|---------|---------|---------|---------|
| Daily wages (Rs.) | 0-50 | 50-100 | 100-150 | 150-200 | 200-250 | 250-300 |
| No. of workers | 1 | 2 | 3 | 4 | 1 | 2 |

Project work

Ask the age of 100 people of your community and present it in the frequency distribution table with a suitable class interval. Present that frequency distribution table in histogram. Find the mean by direct method and short-cut method and present in classroom.

Answers

1. (a) 47 (b) 16 (c) 10.32 (d) 15.05
2. (a) 24 year (b) 43.86 (c) Rs. 706.67 (d) 28.75
3. (a) 20 (b) 818 (c) -100 (d) 35
4. (a) 25 (b) 150 (c) 12 (d) 50
5. (a) 34.5 (b) 31.29 6. (a) 30 (b) Rs. 155.77 (c) 34.5

13.2 Median

Activity 3

The marks obtained in Mathematics by class 10 students in the first terminal examination are given below.

21, 23, 28, 14, 10, 18, 19, 29, 27, 25, 19, 17, 18, 20, 21, 17, 15, 16,
28, 23, 24, 17, 16, 19, 14, 24, 23, 27, 14, 15, 21, 24, 26, 24, 18

On the basis of the above data, solve the following questions and check your answer with your friends

- (a) What is the average mark of the students?
- (b) Find the median value of the student's marks by making individual series and discrete series.
- (c) Does the median value obtained from different series differ?

The median is the statistical value that divides the given data exactly into two parts.

Example 1

The marks obtained by class 8 students in Mathematics are given below. Find the median of the data.

Marks obtained	17	18	22	26	30	32
No. of students	3	4	8	10	7	5

Solution

Here,

Details of the marks obtained by class 8 students

Marks obtained (X)	No. of students (f)	Cumulative frequency (cf)
17	3	3
18	4	7
22	8	15
26	10	25
30	7	32
32	5	37
	$\Sigma f = N = 37$	

We know that,

$$\begin{aligned}\text{The position of median} &= \frac{N+1}{2}^{\text{th}} \text{ term} \\ &= \frac{37+1}{2}^{\text{th}} \text{ term} \\ &= 19^{\text{th}} \text{ term}\end{aligned}$$

From the above table, the value of the 19th item is 26. So, the median of the marks obtained by class 8 students is 26.

Activity 4

We were able to find the median from the data given in discrete series. Now, if the data is given in a continuous series, how to find the median?

Median from the continuous series of data

Like,

Marks obtained (X)	0-8	8-16	16-24	24-32	32-40	40-48	48-56
No. of students (f)	6	10	16	18	12	10	8

The median of the continuous series of data can be found in the following steps:

- Make a less than cumulative frequency table. (Less than the upper value of each class interval)
- Identification of the position of the median. The position of median = $\frac{N}{2}$ th item
- Finding the class interval of median. The class interval of the position of median is the median class interval.

(d) Finding the median by using the following formula.

$$\text{Median } (M_d) = L + \frac{\frac{N}{2} - cf}{f} \times h$$

Where, L = Lower limit of the median class

N = Total numbers of data

cf = Cumulative frequency of the class preceding the median class

f = Frequency of the median class

h = Length (width) of class interval

Here, constructing the table for the median,

Marks obtained (X)	No. of students (f)	Marks	Less than cumulative frequency (cf)
0-8	6	Less than 8	6
8-16	10	Less than 16	6 + 10 = 16
16-24	16	Less than 24	16 + 16 = 32
24-32	18	Less than 32	32 + 18 = 50
32-40	12	Less than 40	50 + 12 = 62
40-48	10	Less than 48	62 + 10 = 72
48-56	8	Less than 56	72 + 8 = 80
	N = 80		

Total number of students (N) = 80

The position of median = $\frac{N}{2}$ th class = $\frac{80}{2}$ th class = 40 th class

40th item lies in the class interval (24 - 32).

Now, the lower limit of the median class (L) = 24

Cumulative frequency of the class preceding the median class (cf) = 32

Frequency of the median class (f) = 18

Length of class interval (h) = 32 - 24 = 8

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 24 + \frac{40 - 32}{18} \times 8$$

$$= 24 + \frac{64}{18}$$

$$= 24 + 3.56 = 27.56$$

Example 2

Find the median from the given data

Weight (Kg)	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	16	12	10	16	18	12

Solution: Here,

Details of the students' weight

Marks obtained (X)	No. of students (f)	Less than cumulative frequency	Cumulative frequency
20 - 30	16	Less than 30 = 16	16
30-40	12	Less than 40 = 16 + 12	28
40-50	10	Less than 50 = 16 + 12 + 10	38
50-60	16	Less than 60 = 16 + 12 + 10 + 16	54
60-70	18	Less than 70 = 16 + 12 + 10 + 16 + 18	72
70-80	12	Less than 80 = 16 + 12 + 10 + 16 + 18 + 12	84
	$\sum f = N = 84$		

Total number of students (N) = 84

The position of median = $\frac{N}{2}$ th item = $\frac{84}{2}$ th item = 42 th item

42th item lies in the class interval (50 - 60).

Now, the lower limit of the median class (L) = 50

Cumulative frequency of the class preceding the median class (cf) = 38

Frequency of the median class (f) = 16

Length of class interval (h) = 60 – 50 = 10

$$\begin{aligned}
 \text{We know that, median } (M_d) &= L + \frac{\frac{N}{2} - cf}{f} \times h \\
 &= 50 + \frac{42 - 38}{16} \times 10 \\
 &= 50 + \frac{40}{60} \\
 &= 50 + 2.5 = 52.5
 \end{aligned}$$

Example 3

The table given below is prepared based on the number of people voluntarily contributing to the public works in the village. If the median value of the given data is 93.6, find the value of the missing frequency 'y'.

Days (X)	0-30	30-60	60-90	90-120	120-150	150-180
No. of workers (f)	5	y	22	25	14	4

Solution: Here,

Table to find the cumulative frequency

Days (X)	No. of workers (f)	Less than cumulative frequency (cf)
0 - 30	5	5
30-60	y	5 + y
60-90	22	27 + y
90-120	25	52 + y
120-150	14	66 + y
150-180	4	70 + y
	N = (70 + y)	

Total number of workers $N = 70 + y$

Median $(M_d) = 93.6$

Median lies in the class interval (90-120)

Now, the lower limit of the median class (L) = 90

Cumulative frequency of the class preceding the median class (cf) = 27 + y

Frequency of the median class (f) = 25

Length of median class interval (h) = 120 - 90 = 30

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$\text{or, } 93.6 = 90 + \frac{\frac{70+y}{2} - (27+y)}{25} \times 30$$

$$\text{or, } 93.6 - 90 = \frac{70+y-2(27+y)}{2 \times 25} \times 30$$

$$\text{or, } 3.6 = \frac{70+y-54-2y}{50} \times 30$$

$$\text{or, } 3.6 = \frac{(16-y) \times 3}{5}$$

$$\text{or, } 3.6 \times 5 = 48 - 3y$$

$$\text{or, } 3y = 48 - 18$$

$$\text{or, } 3y = 30$$

$$\text{or, } y = 10$$

Hence, missing frequency (y) = 10

Activity 5

Below given is a frequency distribution table based on the height of trees in a garden.

Height (ft)	4-6	7-9	10-12	13-15	16-18	19- 21	22-24
No. of trees	2	3	10	7	4	3	2

(a) How to make the above data into continuous series?

(b) What is the median of the above data? Find.

In the data given here, the class intervals are not continuous. So, to make the class interval continuous, the correction factor should be found as follows.

$$\begin{aligned} \text{Correction factor} &= \frac{\text{Lower limit of the second class interval} - \text{Upper limit of the first class interval}}{} \\ &= \frac{7 - 6}{2} = 0.5 \end{aligned}$$

The correction factor is subtracted from the lower value and added to the upper value of each class interval to convert the class interval into continuous series.

Like, in the class interval = 4 - 6

Lower value 4 - 0.5 = 3.5 and upper value 6 + 0.5 = 6.5

Now, we should make class interval 3.5 - 6.5

Table to find the median

Height (cm) X	No. of trees (f)	Less than cumulative frequency (cf)
3.5 - 6.5	2	2
6.5 - 9.5	3	5
9.5 - 12.5	10	15
12.5 - 15.5	7	22
15.5 - 18.5	4	26
18.5 - 21.5	3	29
21.5 - 24.5	2	31
	N = 31	

The position of median = $\frac{N}{2}$ class

$$= \frac{31}{2} = 15.5^{\text{th}} \text{ class}$$

The class interval having 15.5th term = (12.5 - 15.5)

Now, the lower limit of the median class (L) = 12.5

Cumulative frequency of the class preceding the median class (cf) = 15

Frequency of the median class (f) = 7

Length of the median class interval (h) = 15.5 - 12.5 = 3

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 12.5 + \frac{15.5 - 15}{7} \times 3$$

$$= 12.5 + \frac{0.5 \times 3}{7}$$

$$= 12.5 + \frac{1.5}{7}$$

$$= 12.5 + 0.21 = 12.71$$

∴ Hence, the median height of trees = 12.71 ft.

Example 5

Construct a frequency distribution table with a class interval of 10 from the given data and find the median.

21, 9, 34, 42, 17, 54, 13, 38, 23, 39, 49, 29, 38, 44, 21, 42, 19, 7, 29, 8, 55, 36, 39, 13.

Solution**Frequency distribution table**

Class interval (X)	Tally Bars	Frequency f	Less than cumulative frequency cf
0-10		3	3
10-20		4	3+4=7
20-30		5	7+5=12
30-40		6	12+6=18
40-50		4	18+4=22
50-60		2	22+2=24
		N = 24	

The position of median = $(\frac{N}{2})^{\text{th}}$ class

$$= (\frac{24}{2})^{\text{th}} = 12^{\text{th}} \text{ item}$$

The class interval having 12th term = (20 – 30)

Now, the lower limit of the median class (L) = 20

Cumulative frequency of the class preceding to median class (cf) = 7

Frequency of the median class (f) = 5

Length of median class interval (h) = 30 - 20 = 10

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 20 + \frac{12-7}{5} \times 10$$

$$= 20 + 10$$

$$= 30$$

Exercise 13.2**1. Find the median from the given data.**

(a) 2.5, 4.5, 3.6, 4.9, 5.4, 2.9, 3.1, 4.2, 4.6, 2.2, 1.5

(b) 100, 105, 104, 197, 97, 108, 120, 148, 144, 190, 148, 22, 169, 171, 92, 100

(c)

Marks obtained	18	25	28	29	34	40	44	46
No. of students	3	6	5	7	8	12	5	4

(d)

Class interval (x)	102	105	125	140	170	190	200
Frequency (f)	10	18	22	25	15	12	8

2. Find the median from the given data.

(a)

Weight (Kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

(b)

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
Frequency	2	5	8	10	7	5	3

(c)

Expenditure (per day) Rs	less than 100	100-200	200-300	300-400	400-500	more than 500
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	less than 40	less than 60	less than 80	less than 100
No. of students	21	44	66	79	90

3. Find the missing frequency from the data given below.

(a) Median = 35

Marks obtained	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	k	4	5

(b) Median = 132.5

Wages (Rs.)	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	p	4	7	5

(c) Median = 39

Age (yrs)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	50	70	100	300	?	220	70	60

4. Find the median of the following data.

(a)

Marks obtained	50-60	60-70	70-80	80-90	90-100
No. of students	2	3	6	5	4

(b)

Marks obtained	< 20	< 40	< 50	< 80	< 100
No. of students	9	23	43	55	60

(c)

Income (Rs)	< 600	< 700	< 800	< 900	< 1000
No. of workers	30	98	152	177	200

(d)

Temp (°c)	0-9	10-19	20-29	30-39	40-49
Days	8	10	20	15	7

5. (a) The marks obtained by 30 students in a class test are as follows.
22, 56, 62, 37, 48, 30, 58, 42, 29, 39, 37, 50, 38, 41, 32, 20, 28, 16, 43,
18, 40, 52, 44, 27, 35, 45, 36, 49, 55, 40

Construct a frequency distribution table with the class interval of 10 from the above data and find the mean median.

- (b) The height of 40 students of class 10 in cm is given below. Construct a frequency distribution table with the class interval of 5 from the data and find the mean and median.

142, 145, 151, 157, 159, 160, 165, 162, 156, 158, 155, 141, 147, 149,
148, 159, 154, 155, 166, 168, 169, 172, 174, 173, 176, 161, 164, 163,
149, 150, 154, 153, 152, 164, 158, 159, 162, 157, 156, 155

Project work

Ask the ages of 100 people in your community. Find the median age by presenting the obtained data with a class interval of 10.

Answer

- | | | | |
|----------------|-----------------|---------------|----------|
| 1. (a) 3.6 | (b) 121 | (c) 34 | (d) 140 |
| 2. (a) 64.5 kg | (b) 157.5cm | (c) 246.15 lb | (d) 40.9 |
| 3. (a) 6 | (b) 3 | (c) 150 | |
| 4. (a) 78.33 | (b) 47 | (c) 703.70 | (d) 25.5 |
| 5. (a) 39, 40 | (b) 158, 155.42 | | |

13.3 Mode

Activity 6

The temperature of 20 days of a city is as follows find which temperature is repeated maximum.

70, 76, 76, 74, 70, 70, 72, 74, 78, 80, 74, 74, 78, 76, 78, 76, 74, 78, 80, 76

The maximum number of times repeated value of the given data is called mode.

Activity 7

Mode from continuous series

We can find the mode of grouped data in the following steps:

- Since, the number of repeated values is the mode, first find the class interval with the highest frequency.
- Finding the frequency of the model class is f_1 , the frequency of the class preceding the model class is f_0 , the frequency of the class succeeding the model class is f_2 .
- Finding the width of mode l class interval
- The following formula is using to find the mode

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where, L = Lower limit of the model class

f_1 = Frequency of the model class

f_0 = Frequency of the class preceding the model class

f_2 = Frequency of the class succeeding the model class

h = Width (size) of model class interval

Example 1

Find the mode from the data given below.

Weight (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

Solution: Here,

Weight (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

Here, the highest frequency is 11, so its corresponding class is 60 - 70.

Lower limit of the model class (L) = 60

Frequency of the model class (f_1) = 11

Frequency of the class preceding the model class (f_0) = 7

Frequency of the class succeeding the model class (f_2) = 10

Width (size) of model class interval (h) = 70 - 60 = 10

We know that, mode (Mo) = $L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$= 60 + \frac{11 - 7}{2 \times 11 - 7 - 10} \times 10$$

$$= 60 + \frac{4}{5} \times 10$$

$$= 60 + 8 = 68$$

Hence, mode (Mo) = 68

Exercise 13.3

1. Find the mode of the following data

(a) 29 cm, 34 cm, 29 cm, 26 cm, 55 cm, 34 cm, 35 cm, 40 cm, 34 cm, 56 cm

(b) 99 kg, 135 kg, 182 kg, 49 kg, 189 kg, 196 kg, 78 kg, 192 kg, 182 kg

2. Find the mode from given frequency table

(a)

Marks obtained	5	10	15	20	25	30	35	40	45
No. of students	2	6	7	9	11	5	15	2	3

(b)

Wages (Rs.)	50	75	100	125	150	175	200	225
No. of workers	8	12	17	29	30	27	20	11

3. Find the mode from the given frequency table

(a)

Marks obtained.	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	6	4	5

(b)

Wages (Rs.)	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	4	7	5	4

(c)

Age (yrs)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	50	70	100	300	220	150	70	60

Answer

- (a) 34 cm (b) 182 kg
- (a) 35 (b) 150 kg
- (a) 33 (b) 136 (c) 38.57

13.4 Quartiles

Activity 8

In class 9, we studied to find the first quartile and third quartile from the individual and discrete data. The data given below is the marks obtained in Mathematics by class 10 students of Janta Secondary School in the first terminal examination.

21, 23, 28, 14, 10, 18, 19, 29, 27, 25, 19, 17, 18, 20, 21, 17, 15, 16,

28, 23, 24, 17, 16, 19, 14, 24, 23, 27, 14, 15, 21, 24, 26, 24, 18

From the above data, find the value of the first quartile and the third quartile by making individual series and discrete series. Is the value of the first quartile and the third quartile obtained from different series also different? Discuss in the group of two.

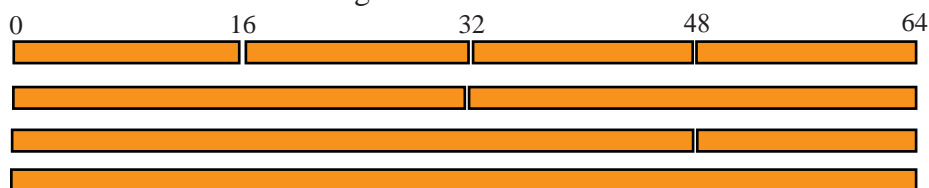
Activity 9

How to find the first quartile and the third quartile from the data given in continuous series? Do the following activities in groups:

- (a) Take any 4 sticks of length 64 cm.

- (b) Divide first stick into 4 equal parts. Find the length of each piece. 16 divide by 64 into 4 equal parts. This is the first quartile.
- (c) Let's divide the second stick into 2 equal parts and find the length of each piece.
- (d) Divide the third stick into 4 equal parts such that 3 parts in one side and one part on another side.

This can be shown in the figure as follows.



As seen in the picture above, when divided into 4 equal pieces, 16 cm pieces are formed. When divided into 2 equal parts, 32 cm pieces are formed. It is also called median. Similarly, if you add 3 equal pieces out of 4 equal pieces, the total length is 48 cm. 16 is the first quartile (Q_1), 32 is the second quartile (Q_2) and 48 is the third quartile (Q_3).

For grouped series

- (a) Find the less than cumulative frequency.
- (b) The position of first quartile $Q_1 = \frac{N}{4}^{\text{th}}$ item and the position of the third quartile $Q_3 = (\frac{3N}{4})^{\text{th}}$ item
- (c) Look at the cumulative frequency for (Q_1) in the class interval with cumulative frequency equal to or greater than $\frac{N}{4}$ and for (Q_3) in the class interval with cumulative frequency exactly greater than $\frac{3N}{4}$.
- (d) After that, use the following formula:

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times h$$

Here,

L = Lower limit of the first quartile (Q_1) class

N = Total numbers of data

cf = Cumulative frequency of the class preceding the first quartile (Q_1) class

f = Frequency of the first quartile class

h = Length (width) of class interval

$$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times h$$

Where, L = Lower limit of the third quartile class

N = Total numbers of data

cf = Cumulative frequency of the class preceding to the third quartile class

f = Frequency of the third quartile class

h = Length (width) of class interval

Example 1

Find the first quartile (Q_1) and third quartile (Q_3) from the following data.

Age (yrs)	20	25	28	30	32	35	42	46
No. of workers	2	8	12	10	14	7	5	1

Solution

Details of workers' age

Workers age X	No. of workers (f)	cf
20	2	2
25	8	10
28	12	22
30	10	32
32	14	46
35	7	53
42	5	58
46	1	59

$$\begin{aligned}
 \text{The position of the first quartile} &= \frac{N+1}{4}^{\text{th}} \text{ term} \\
 &= \frac{59+1}{4}^{\text{th}} \text{ term} \\
 &= \frac{60}{4} = 15^{\text{th}} \text{ term}
 \end{aligned}$$

The corresponding value of the 15th item is 28. So, the first quartile (Q_1) is 28.

$$\begin{aligned}
 \text{Again, the position of the third quartile} &= \frac{3(N+1)}{4}^{\text{th}} \text{ term,} \\
 &= \frac{3(59+1)}{4}^{\text{th}} \text{ term} = \frac{180}{4} = 45^{\text{th}} \text{ term}
 \end{aligned}$$

The corresponding value of the 45th item is 32. So the third quartile (Q_3) is 32.

Example 2

The marks obtained in Mathematics by class 7 students in are given below. Find the first quartile (Q_1) and the third quartile (Q_3) from the data.

Marks obtained (X)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	2	8	15	14	10	8	3

Solution

Details of the mark obtained by students

Marks obtained (X)	No. of students (f)	Cumulative frequency (cf)
10-20	2	2
20-30	8	10
30-40	15	25
40-50	14	39
50-60	10	49
60-70	8	57
70-80	3	60
$\Sigma f = N = 60$		

The position of the first quartile (Q_1) = $\frac{N}{4}$ th item = $\frac{60}{4}$ th item = 15th item

The class interval having the 15th item is (30 - 40)

Now, lower limit of the first quartile class interval (L) = 30

Cumulative frequency of the class preceding the first quartile class (cf) = 10

Frequency of the first quartile class (f) = 15

Length (width) of first quartile class interval (h) = 40 - 30 = 10

$$\begin{aligned}\text{We know that, first quartile } (Q_1) &= L + \frac{\frac{N}{4} - cf}{f} \times h \\ &= 30 + \frac{15 - 10}{15} \times 10 \\ &= 30 + \frac{50}{15} \\ &= 30 + 3.34 = 33.34\end{aligned}$$

The position of the third quartile = $\frac{3N}{4}$ th item = $\frac{3 \times 60}{4}$ th item = 45th item

The class interval having the 45th item is (50 - 60).

Now, lower limit of the third quartile class interval (L) = 50

Cumulative frequency of the class preceding the third quartile class (cf) = 39

Frequency of the third quartile class (f) = 10

Length (width) of the third quartile class interval (h) = 60 - 50 = 10

$$\begin{aligned}\text{We know that, third quartile } (Q_3) &= L + \frac{\frac{3N}{4} - cf}{f} \times h \\ &= 50 + \frac{45 - 39}{10} \times 10 \\ &= 50 + \frac{60}{10} \\ &= 50 + 6 = 56\end{aligned}$$

Hence, the first quartile (Q_1) = 33.34 and the third quartile (Q_3) = 56

Example 3

In the following table workers' incomes are given. Find Q_1 , Q_2 and Q_3 from the data.

Income (in thousands)	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of workers	10	15	40	55	30	25	5

Solution

Details of workers' income

Income (in thousands) (X)	No. of workers (f)	Cumulative frequency (cf)
0-5	10	10
5-10	15	25
10-15	40	65
15-20	55	120
20-25	30	150
25-30	25	175
30-35	5	180

Total number of workers (N) = 180

The position of the first quartile = $\frac{N}{4}$ th item = $\frac{180}{4}$ th item = 45th item

The class interval having the 45th item is (10 – 15)

Now, lower limit of the first quartile class interval (L) = 10

Cumulative frequency of the class preceding the first quartile class (cf) = 25

Frequency of the first quartile class (f) = 40

Length (width) of the first quartile class interval (h) = 15 - 10 = 5

$$\begin{aligned}\text{We know that, first quartile } (Q_1) &= L + \frac{\frac{N}{4} - cf}{f} \times h \\ &= 10 + \frac{45 - 25}{40} \times 5 \\ &= 10 + \frac{100}{40} \\ &= 10 + 2.5 = 12.5\end{aligned}$$

Again, the position of the second quartile or median = $\frac{N}{2}$ th item = $\frac{180}{2}$ th item = 90th item

The class interval having 90th item is (15 - 20)

Now, lower limit of the median class (L) = 15

Cumulative frequency of the class preceding the median class (cf) = 65

Frequency of the median class (f) = 55

Length (width) of median class interval (h) = 20 - 15 = 5

$$\begin{aligned}\text{We know that, second quartile or median } (Q_2) &= L + \frac{\frac{N}{2} - cf}{f} \times h \\ &= 15 + \frac{90 - 65}{55} \times 5 \\ &= 15 + \frac{25}{11} \\ &= 15 + 2.27 = 17.27\end{aligned}$$

Now, the position of the third quartile = $\frac{3N}{4}$ th item = $\frac{3 \times 180}{4}$ th item = 135th item

The class interval having the 135th item is (20 - 25).

Now, lower limit of the third quartile class (L) = 20

Cumulative frequency of the class preceding the third quartile class (cf) = 120

Frequency of the third quartile class (f) = 30

Length (width) of third quartile class interval (h) = 25 - 20 = 5

We know that, third quartile (Q_3) = $L + \frac{\frac{3N}{4} - cf}{f} \times h$

$$= 20 + \frac{135 - 120}{30} \times 5$$

$$= 20 + \frac{15}{6} = 20 + 2.5$$

$$= 22.5$$

Hence, the first quartile (Q_1) = 12.5, the second quartile or median (Q_2) = 17.27 and the third quartile (Q_3) = 22.5.

Example 4

Students' expenditure on tiffin in one week and the number of students are given in If the upper quartile of the data is 460, then find the value of p.

Expenditure (Rs.)	100-200	200-300	300-400	400-500	500-600
No. of students	15	18	P	20	17

Solution

Table to find quartile

Expenditure (Rs) (X)	No. of students (f)	Cumulative frequency (cf)
100-200	15	15
200-300	18	33
300-400	p	33 + p
400-500	20	53 + p
500-600	17	70 + p
	N = 70 + p	

Total number of students (N) = 70 + p

Since, third quartile (Q_3) = 460

The third quartile class interval is (400 - 500).

Now, lower limit of the third quartile class (L) = 400

Cumulative frequency of the class preceding the third quartile class (cf) = 33 + p

Frequency of the third quartile class (f) = 20

Length (width) of the third quartile class interval (h) = 500 - 400 = 100

We know that, third quartile (Q_3) = $L + \frac{\frac{3N}{4} - cf}{f} \times h$

$$\text{or, } 460 = 400 + \frac{\frac{3(70+p)}{4} - (33+p)}{20} \times 100$$

$$\text{or, } 460 - 400 = \frac{210 + 3p - 132 - 4p}{4 \times 20} \times 100$$

$$\text{or, } 60 = \frac{78 - p}{4} \times 5$$

$$\text{or, } 78 - p = \frac{60 \times 4}{5} = 48$$

$$\text{or, } 78 - 48 = p$$

$$\text{or, } 30 = p$$

$$\therefore p = 30$$

Hence, the missing frequency (p) = 30

Exercise 13.4

1. Find the value of Q_1 and Q_3 from the data given below:

(a) 10, 12, 14, 11, 22, 15, 27, 14, 16, 13, 25

(b)

Marks obtained	42	48	49	53	56	59	60	65	68	70
No. of students	2	3	5	8	9	11	7	8	6	4

2. Find the value of Q_1 and Q_3 from the data given below:

(a)

Age (yrs)	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
No. of students	5	12	25	26	24	28	20	15

(b)

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	12	13	11	7

(c)

Height (cm)	100-110	110-120	120-130	130-140	140-150	150-160	160-170
No. of students	3	4	9	15	20	14	7

(d)

Wages (Rs.)	100-150	150-200	200-250	250-300	300-350	350-400
No. of workers	6	11	21	34	25	22

(e)

Marks obtained	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	8	12	15	14	12	9	10

(f)

Time (minutes)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	5	3	10	6	4	2

(g)

Time (minutes)	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	2	5	8	6	4	5

3. (a) If $Q_1 = 8$, what is the value of k ?

Age(yrs)	0-6	6-12	12-18	18-24	24-30	30-36
No. of pupils	9	6	5	k	7	9

- (b) If $Q_1 = 31$, what is the missing frequency?

Class interval	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	5	?	8	7	6

- (c) If $Q_1 = 51.75$, what is the value of q ?

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64
Frequency	8	10	14	q	3	1

4. Find the value of Q_1 and Q_3 from the data given below:

(a)

Height (cm)	<125	<130	<135	<140	<145	<150	<155
No. of students	0	5	11	24	45	60	72

(b)

Weight (lbs)	110-119	120-129	130-139	140-149	150-159	160-169	170-179	180-189
Frequency	5	7	12	20	16	10	7	3

(c)

Expenditure (per day)	Less than 100	100-200	200-300	300-400	400-500	More than 500
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	less than 40	less than 60	less than 80	less than 100
No. of students	21	44	66	79	90

5. (a) The data given below represent the marks obtained by 30 students in an internal examination. Find the first and third quartiles by tabulating the data taking a class interval of 10.

42, 65, 78, 70, 62, 50, 72, 34, 30, 40, 58, 53, 30, 34, 51, 54, 42, 59, 20, 40, 42, 60, 25, 35, 35, 28, 46, 60, 47, 52

- (b) The number of eggs produced everyday in a chicken farm is given below. Find the first and third quartiles by tabulating the data taking a class interval of 20.

32, 87, 17, 51, 99, 79, 64, 39, 25, 95, 53, 49, 78, 32, 42, 48, 59, 86, 69, 57, 15, 27, 44, 66, 77, 92

Project work

Ask and write the total marks obtained by the 100 students of classes 9 and 10 in an internal examination out of 100 full marks.

- Construct the frequency distribution table with a suitable class interval of the given data.
- Prepare the more than and less than cumulative frequency table by using the given data.
- Prepare a report of all the work in sequential order and present it in the classroom.

Answer

- 12, 22
 - 53, 65
- 7.74, 13.73
 - 42.08, 64.09
 - 131.33, 152.14
 - 230.35, 334.5
 - 40, 98.33
 - 18.3, 37.5
 - 30.31, 40.63
- 8
 - 10
 - 16
- 137.69, 148
 - 136.16, 159.5
 - 152.94, 360
 - 20.45, 61.15
- 37.5, 59.28
 - 38, 61.57