Lesson 6

Sequence and Series

6.0 Review

Observe the following sequences and answer the questions.

(a) 2, 4, 6, 8, …….. (b) 1, 3, 5, 7, …….

(c) 1, 4, 16, 64 ,…….. (d) 24, 12, 6, 3, …….

(e) 5, 11, 8, 14 …….. (f) 2, 6, 18, 54 , …….

Questions

(i) Identify the above sequences whether they are arithmetic or geometric.

(ii) Find the common difference or common ratio of the sequences.

(iii) Find the general term of the sequences.

(iv) Find the sixth and eight terms of the sequences.

(v) Express the sequences into series.

6.1 Means of Arithmetic Sequence

Activity 1

Study the following conversation.

Rita: How can the value of m be found, if 3, m, 7, ……are in arithmetic sequences?

Raju: The difference of each term to its preceding term of an arithmetic sequence is equal.

So, $m - 3 = 7 - m$

$m - 3 = 7 - m$

or, $m + m = 7 + 3$

or, $m = \frac{7+3}{2} = 5$

Rita: Yes. The second term is the average of the first and the third terms.

Anil: It can also be solved as follows.

First term $(a) = 3$

Second term $(t_2) = a + d$, where d is the common difference.

Third term $(t_3) = a + 2d$

or, $7 = 3 + 2d$

or, $2d = 7 - 3$

or, $2d = 4$

$\therefore d = 2$

Putting the value of d in $t_2 = a + d$

$m = 3 + 2 = 5$

Rashmi : How can we find more than 1 unknown terms between two given terms?

For example: What are the values of x, y, z in the sequence $3, x, y, z, 19$.....?

Anil : It can be solved In the following way.

First term (a) = 3

Fifth term $(t_5) = 19$

Common difference (d) = ?

We know that,

$t_5 = a + (5-1)d$

or, $19 = 3 + 4d$

or, $4d = 19 - 3$

or, $4d = 16$

$\therefore d = 4$

Second term $(t_2) = x = a + d = 3 + 4 = 7$

Third term $(t_3) = y = a + 2d = 3 + 2 \times 4 = 11$

Fourth term $(t_4) = z = a + 3d = 3 + 3 \times 4 = 15$

Rita : Yes, I knew that the three terms between 3 and 19 can be found taking 19 as the fifth term.

Anil : Yes, you are correct.

Total number of terms = number of means + 2

$$n = m + 2$$

(Finally, they reported their discussion to the mathematics teacher.)

Teacher: You are all correct.

The terms lying between the first and the last terms of an arithmetic sequence are called arithmetic means.

(a) An arithmetic mean between the first and the last terms can be obtained by taking the arithmetic average of the first and the last terms.

(b) If there are more than one arithmetic means between two terms, they can be obtained by calculating common difference (d).

Example 1

Find an arithmetic mean between the numbers 7 and 17.

Solution

Let m be the arithmetic mean between 7 and 17.

Then, 7, m, 17 are in arithmetic sequence.

First term (a)=7

Third term $(t_3) = 17$

Arithmetic mean (m) = ?

We know that $(m) = \frac{a+b}{2}$

$= \frac{7+17}{2}$

$=12$

Alternative method

Third term $(t_3) = a + 2d$

or, $17 = 7 + 2d$

or, $2d = 17 - 7 = 10$

or, $d = 5$

Now, $(t_2) = a + d = 7 + 5 = 12$

$\therefore$ m = 12

Example 2

Find three arithmetic means between the terms 3 and 23.

Solution

The sequence with three arithmetic means is 3, m_1, m_2, m_3, 23

First term (a) = 3

Number of means (m) = 3

Total number of terms (n) = 3 + 2 = 5

Fifth term $(t_5) = 23$

We know that

Fifth term $(t_5) = a + (5 - 1)d$

or, $23 = 3 + 4d$

or, $23 - 3 = 4d$

or, $\frac{20}{4} = d$

$\therefore\ d = 5$

Now, m_1 = second term $= a + d = 3 + 5 = 8$

m_2 = third term = $a + 2d = 3 + 2 \times 5 = 13$

m_3 = fourth term = $a + 3d = 3 + 3 \times 5 = 18$

Hence, the three arithmetic means between 3 and 23 are 8, 13 and 18.

Example 3

Find 7 arithmetic means between two terms 3 and 43.

Solution

The arithmetic sequence with 7 means is $3, m_1, m_2, m_3, m_4, m_5, m_6\ m_7, 43$.

First term $(a) = 3$

Number of means $(m) = 7$

Total number of terms $(n) = 7 + 2 = 9$

Ninth term $(t_9) = 23$

We know, ninth term $(t_9) = a + (9 - 1)d$

or, $43 = 3 + 8d$

or, $8d = 43 - 3 = 40$

or, $d = 5$

Now, m_1 = second term $= a + d = 3 + 5 = 8$

m_2 = third term = $a + 2d = 3 + 2 \times 5 = 13$

m_3 = fourth term = $a + 3d = 3 + 3 \times 5 = 18$

m_4 = fifth term $= a + 4d = 3 + 4 \times 5 = 23$

m_5 = sixth term = $a + 5d = 3 + 5 \times 5 = 28$

m_6 = seventh term = $a + 6d = 3 + 6 \times 5 = 33$

m_7 = eighth term $= a + 7d = 3 + 7 \times 5 = 38$

6.2 Sum of Arithmetic Series

Activity 2

A shopping centre provides cash incentive to its employees according to their performance. One of the employees receives cash increment in his salary as given in the following table.

Year 2079	Baisakh	Jestha	Ashadh	Shrawan	Bhadra	Ashoj	Kartik
Increment in Salary (Rs.)	800	1000	1200	1400	1600	1800	2000

Study the information given in table and discuss the following questions.

(a) How much is the incentive amount increasing every month?

(b) What is the total amount of incentives received from Baisakh to Kartik of 2079 BS.?

(c) What will be the amount of the incentive in the month of Baisakh, 2080 B.S. if the same rate of growth continues?

(d) What will be the total amount of incentives that an employee receives upto Baisakh , 2080 BS?

The incentive amount is increased by Rs. 200 every month. The series of incentive amounts of 7 months is 800 + 1000 + 1200 + 1400 + 1600 + 1800 + 2000.

If the total amount of incentives in seven months is denoted by S_7,

$$S_7 = 800 + 1000 + 1200 + 1400 + 1600 + 1800 + 2000 = 9800 \ldots\ldots \text{(i)}$$

First term (a) = 800, Common difference =1000 − 800 = 200

Number of term (n) = 7, seventh term (t_7) = 2000

The equation (i) can be expressed as

$$2S_7 = 2000 + 1800 + 1600 + 1400 + 1200 + 1000 + 800 = 9800 \ldots\ldots \text{(ii)}$$

Adding the equations (i) and (ii),

$$S_7 = 2800 + 2800 + 2800 + 2800 + 2800 + 2800 + 2800$$

$$\text{or, } 2S_7 = 7 \times 2800$$

$$\text{or, } S_7 = \frac{7}{2} \times 2800$$

$$= 9800$$

Expressing the value 2800 in terms of first and seventh terms,

or, $S_7 = \frac{7}{2}(800 + 2000)$

$\therefore S_n = \frac{n}{2}(a + t_n)$ where a= incentive amount of first month, t_n = incentive amount of last month.

Now, $S_n = \frac{n}{2}(a + t_n)$

$= \frac{n}{2}[a + a + (n-1)d] = \frac{n}{2}[2a + (n-1)d]$ $[\because t_n = a + (n-1)d]$

Total number of months upto Baisakh, 2080 BS is 13

Incentive amount of 13th month $(t_{13}) = 800 + (13 - 1)\,200$

$= 800 + 12 \times 200$

$=$ Rs. 3,200

Hence, the incentive amount received in Baisakh, 2080 BS is Rs. 3200.

The total amount of incentive received in 13 months $(S_{13}) = ?$

We know that,

$S_n = \frac{n}{2}[(2a + (n-1)d]$

$= \frac{13}{2}[2 \times 800 + (13 - 1)\,200]$

$= \frac{13}{2}[1600 + (12 \times 200] = \frac{13}{2}[1600 + 2400] = \frac{13}{2} \times 4000$

$= 26000$

Hence, the total amount of incentive received in 13 months = Rs. 26,000.

The sum of the first n terms of an arithmetic series is $S_n = \frac{n}{2}(a + t_n)$,
where first term = a, final term = t_n, common difference = d,
number of terms = n, and sum of the first n terms = S_n

Again, substituting $t_n = a + (n-1)d$

$S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}[a + \{a + (n-1)d\}] = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

Note : The final term can be denoted be either (t_n) or l.

Example 4

Find the sum of the first 20 terms of the series 1 + 2 + 3 + . . .

Solution

Here, 1 + 2 + 3 + . . . is an arithmetic series.

First term (a) = 1

Common difference (d) = 2 – 1 = 1

Number of terms (n) = 20

Sum of the first 20 terms (S_{20}) = ?

We know that,

$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{20}{2}[2 \times 1 + (20-1)1]$$
$$= 10(2 + 19)$$
$$= 10 \times 21 = 210$$

Example 5

Find the sum of the first 13 terms of the series 2 + 4 + 6 + …

Solution

Here, 2 + 4 + 6 + …… is an arithmetic series.

First term (a) = 2

Common difference (d) = 4 – 2 = 2

Number of terms (n) = 13

Sum of the first 13 terms (S_{13}) = ?

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{13}{2}[2 \times 2 + (13-1)2]$$
$$= \frac{13}{2}(4 + 24)$$
$$= \frac{13}{2} \times 28$$
$$= 182$$

Example 6

Find the sum of the first 28 terms of the series 5 + 17 + 29 +…

Solution

Here, 5+17+29+……is an arithmetic series.

First term $(a) = 5$

Common difference $(d) = 17 - 5 = 12$

Number of terms $(n) = 28$

Sum of the first 28 terms $(S_{28}) = ?$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{28} = \frac{28}{2}[2 \times 5 + (28 - 1)12]$

$= 14(10 + 324)$

$= 14 \times 334 = 4676$

Example 7

An arithmetic series has the first term 2, the final term 29 and sum of its first n terms is 155. Find the number of terms and the common difference of the series.

Solution

First term $(a) = 2$,

Final term $(t_n) = 29$ and,

Sum of first n terms $S_n = 155$

Number of the terms $(n) = ?$

Common difference $(d) = ?$

We know that $S_n = \frac{n}{2}(a + t_n)$

or, $155 = \frac{n}{2}(2 + 29)$

or, $310 = 31n$

or, $n = \frac{310}{31} = 10$

Again, we have,

$t_n = a + (n-1)d$

or, $29 = 2 + (10-1)d$

or, $27 = 9d$

or, $d = \frac{27}{9} = 3$

Hence, the common difference is 3.

Example 8

If the sum of certain number of terms in the arithmetic series 24+20+16+…… is 72, find the number of terms.

Solution

Here, first term (a) = 24

Common difference (d) = 20 – 24 = – 4

Sum of first n terms $S_n = 72$ छ

Number of terms (n) = ?

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$72 = \frac{n}{2}[2 \times 24 + (n-1)(-4)]$

or, $144 = n(48 - 4n + 4)$

or, $144 = 52n - 4n^2$

or, $4n^2 - 52n + 144 = 0$

or, $n^2 - 13n + 36 = 0$

or, $n^2 - 9n - 4n + 36 = 0$

or, $n(n-9) - 4(n-9) = 0$

or, $(n-9)(n-4) = 0$

either, $n - 9 = 0 \quad \therefore n = 9$

or, $n - 4 = 0 \quad \therefore n = 4$

Hence the total number of terms is either 4 or 9.

Example 9

Find the sum of the first 15 terms of an arithmetic series if its third term is 0 and the tenth term is 42.

Solution

Here, third term $(t_3) = 0$

Tenth term $(t_{10}) = 42$

Total number of terms $(n) = 15$

Sum of first 15 terms $S_{15} = ?$

We know that,

$t_n = a + (n - 1)\, d$

or, $t_3 = a + (3 - 1)\, d$

or, $0 = a + 2d$

or, $a + 2d = 0$ ..... (i)

Also, $t_{10} = a + (10 - 1)\, d$

or, $42 = a + 9d$

$a + 9d = 42$ ..... (ii)

Subtracting the equation (ii) from (i)

$$\begin{array}{r} a + 2d = 0 \\ \underline{a + 9d = 42} \\ -7d = -42 \end{array}$$

or, $d = \frac{42}{7} = 6$

$\therefore d = 6$

Substituting the value of d in equation (i),

$a + 2 \times 6 = 0$

$a = -12$

Now, sum of the first 15 terms $S_{15} = \frac{15}{2}[2 \times (-12) + (15 - 1)6]$

$= \frac{15}{2}[-24 + 84]$

$= 450$

Hence, the sum of the first 15 terms is 450.

Example 10

The salary of a person is Rs. 40,000 at present and he receives a grade of Rs 1,000 on his monthly salary every year. What will be his total income in 6 years if the number of grades goes on increasing as the number of years increases? How much additional amount is required to earn Rs. 35,00,000 in 6 years?

Solution

Here, total income of the first year $(a) = 40000 \times 12 = 4{,}80{,}000$

Increased amount every year $(d) = 1000 \times 12 = 12{,}000$

Total income in 6 years $(S_6) = ?$

Number of years $(n) = 6$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = \frac{6}{2}[2 \times 480000 + (6-1) \times 12000]$$

$$= 3[960000 + 60000]$$

$$= 3 \times 10{,}20{,}000$$

$$= 30{,}60{,}000$$

Hence, the total income in 6 years is Rs. 30,60,000.

To earn Rs. 35,00,000 in 6 years, the required amount $= 35{,}00{,}000 - 30{,}60{,}000 = 4{,}40{,}000$.

Exercise 6.1

1. (a) What are arithmetic means?

(b) If a, m, b are in arithmetic sequence, express m in terms of a and b.

(c) Find the mid value of the numbers 12 and 18.

2. Find the arithmetic mean between

(a) 6 and 10 (b) –2 and 2 (c) –4 and 8 (d) $(a + b)$ and $(a - b)$

3. Find arithmetic means in the following cases.

(a) 4 arithmetic means between 5 and 20

(b) 6 arithmetic means between 70 and 14

(c) 6 arithmetic means between 5 and -9

4. The following sequences are arithmetic sequences. Fom the sequences, find the value of x.

(a) $5, x, 9, \ldots$

(b) $x + 1, x + 5, 3x + 1, \ldots$

(c) $x + 2, 3x, 4x + 1, \ldots$

5. Find the sum of the following arithmetic series.

(a) $7 + 11 + 15 + 19\ldots$, to 20 terms

(b) $4 - 1 - 6 - 11 - 16 - \ldots$, to 7 terms

(c) $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \ldots$, to 16 terms

(d) $5 + 10 + 15 + \ldots + 65$

(e) $-64 - 48 - 32 - \ldots + 32$

(f) The sum of first 10 odd numbers

(g) The sum of first 100 natural numbers

(h) The sum of natural numbers from 50 to 100

6. The first term, the last term and the sum of all the terms of an arithmetic series are given below. Find the common difference of the series.

(a) First term = 1, last term = 50 and sum of all terms = 204

(b) First term = 160, last term = 20 and sum of all terms = 440

(c) First term = 17, last term = $-\frac{99}{8}$ and sum of all terms = $\frac{407}{16}$

7. The common difference, number of terms and the sum of all the terms of an arithmetic series are given as follows. Find its first term.

(a) Common difference = -3, number of terms = 10 and sum of all terms = 325

(b) Common difference = 9, number of terms = 9 and sum of all terms = 108

(c) Common difference = 3, number of terms = 10 and sum of all terms = 155

8. (a) How many terms of the series $4 + 10 + 16 + 22 + \ldots\ldots\ldots\ldots$ has a sum 374?

(b) If the first term of an arithmetic series is 36 and its common difference is 9, how many terms will have a sum 540?

9. (a) Find the sum of the first 16 terms of an arithmetic series , if its third term is -15 and eighth term is 10.

(b) Find the sum of the first 20 terms of an arithmetic series , if its fifth term is 10 and eleventh term is 22.

10. (a) If the sum of the first 6 terms of an arithmetic series is 75 and the sum of its first 12 terms is 390, find the sum of the first 20 terms.

(b) If the sum of the first 7 terms of an arithmetic series is 21 and the sum of its first 12 terms is 126, then find the series.

11. (a) A worker in a garment factory produced 1000 Nepali cap (Dhaka Topi) at the first year and increased the production by 100 every year. Can he produce 15,000 caps in 10 years?

(b) The monthly salary of a person appointed for a new job is Rs. 60,000 initially and it increases by Rs. 24,000 every year. In how many years he will earn the total amount of Rs.1 crore ? (The interest is not included in his earning)

Project work

Request your parents to deposit some amount on the occasion of your birthday and continue it for 5 years in such a way that the deposit amount for every year increases by the same value. Calculate the total amount deposited in 5 years and compare who will collect more amount in your group of classmates.

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1. (b) $m = \frac{a+b}{2}$ (c) 15 2. (a) 8 (b) 0 (c) 0 (d) a
3. (a) 8, 11, 14, 17 (b) 62, 54, 46, 38, 30, 22 (c) 3, 1, –1, –3, –5, –7
4. (a) 7 (b) 4 (c) 3
5. (a) 900 (b) –77 (c) 128 (d) 455
   (e) –112 (f) 100 (g) 5050 (h) 3825
6. (a) 7 (b) –4 (c) $-\frac{47}{16}$
7. (a) 46 (b) –24 (c) 2
8. (a) 11 (b) 8
9. (a) 200 (b) 420
10. (a) 1050 (b) –6 –3 + 0 +…
11. (a) No, less by 500 (12) Nearly 12 years

6.3 Means of Geometric Sequence

Activity 1

How can the unknown terms of the following geometric sequences be found? Discuss the solution in each of your pairs and present the conclusion.

(a) 3, m, 27, . . .

(b) 2, m_1, m_2, m_3, 512, . . .

(a) Since 3, m, 27,…….. are in geometric sequence, $\frac{3}{m} = \frac{m}{27}$

$\frac{3}{m} = \frac{m}{27}$

or, $m^2 = 3 \times 27$

or, $m = \sqrt{(3 \times 27)} = 9$

Second term $= \sqrt{\text{first term} \times \text{third term}}$

(b) 2, m_1, m_2, m_3, 512, . . .are in geometric sequence.

First term (a) = 2

Number of means (m) = 3

Total number of terms (n) = $m + 2 = 3 + 2 = 5$

Fifth term (t_5) = 512

We know that, $t_n = ar^{n-1}$

or, $512 = 2r^{5-1}$

or, $256 = r^4$

or, $(4)^4 = r^4$

$\therefore r = 4$

Then, second term (t_2) = $m_1 = ar = 2 \times 4 = 8$

Third term (t_3) = $m_2 = ar^2 = 2 \times 4^2 = 32$

Fourth term (t_4) = $m_3 = ar^3 = 2 \times 4^3 = 128$

The terms lying between the first and last terms of a geometric sequence are called geometric means.

(a) The single geometric mean between two terms is the square root of their product.

(b) Two or more than two geometric means can be calculated using the first term and common ratio. The common ratio of the sequence can be found taking the last term as n^{th} term.

Example 1

Find a geometric mean between two numbers 2 and 32.

Solution

If m is the geometric mean between 2 and 32, then the numbers 2 m, and 32 are in geometric sequence.

First term (a) = 2

Third term (t_3) = 32

Mean (m) = second term =?

We know, mean (m) = $\sqrt{(a \times t_3)} = \sqrt{(2 \times 32)} = 8$

Hence, the geometric mean (m) = 8.

Example 2

Find 3 geometric means between two numbers, 5 and 405.

Solution

If m_1, m_2, m_3 are the 3 geometric means between 5 and 405, then 5, m_1, m_2, m_3, 405

First term (a) = 5

Number of means (m) = 3

Total number of terms (n) = 3 + 2 = 5

Fifth term (t_5) = 405

We know that $t_5 = ar^{5-1}$

or, $405 = 5r^{5-1}$

or, $405 = 5r^4$

or, $81 = r^4$

or, $3^4 = r^4$

$\therefore r = 3$

Then, m_1 = second term $= ar = 5 \times 3 = 15$

m_2 = third term $= ar^2 = 5 \times 3^2 = 45$

m_3 = fourth term $= ar^3 = 5 \times 3^3 = 135$

Hence, 3 geometric means between 5 and 405 are 15, 45 and 135.

6.4 Sum of Geometric Series

Activity 2

An employee working in a pharmaceutical industry had a salary of Rs. 10,000 initially. Because of the pandemic of Covid 19, the workers were highly demanded and it was decided to increases their salary by 10% every month. How much will he earn in 5 months if the salary increased by 10% every month? Discuss the problem.

Here, the monthly salary at the beginning =1000

Salary of the second month =10,000 + 10,000 × 10% = 11,000

Salary of the third month = 11,000 + 11,000 × 10% = 12,100

Salary of the fourth month = 12,100 + 12,100 × 10% = 13,310

Salary of the fifth month = 13,110 + 13,110 × 10% = 14,641

Now, the salary of 5 months in series is

$S_5 = 10000 + 11000 + 12100 + 13310 + 14641$……(i)

Common ratio (r) $= \frac{11000}{10000} = \frac{12100}{11000} = \frac{13310}{12100} = \frac{14641}{13310} = 1.1$

Multiplying the equation (i) by 1.1

$1.1 \times (S_5) = 11000 + 12100 + 13310 + 14641 + 16105.1$……(ii)

Subtracting the equation (i) from (ii),

or, $(1.1 - 1)\, S_5 = 16105.1 - 10000$

or, $(1.1 - 1)\, S_5 = 10000\,(1.1)^5 - 10000$

or, $(1.1 - 1)\, S_5 = 10000\{(1.1)^5 - 1\}$

0r, $S_5 = \frac{10000[(1.1)^5 - 1]}{(1.1 - 4)}$

It is in the form of $S_n = \frac{a(r^n - 1)}{(r - 1)}$ where a is the first term, r is the common ratio and n is the total number of terms.

or, $S_5 = 61051$

Hence, the earning of the employee in 5 months is Rs. 61051.

The sum of the first n terms of a geometric sequence is given by $S_n = \frac{a(r^n - 1)}{(r - 1)}$ where r >1 and $S_n = \frac{a(1 - r^n)}{(1 - r)}$ जहाँ $r < 1$.

Here, first term = a, common ratio = r, number of terms = n and the sum of first n terms = S_n.

Again, $S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{ar^n - a}{(r - 1)} = \frac{ar^{n-1} \times r - a}{(r - 1)}$

$= \frac{t_n\, r - a}{r - 1}$ $\quad \because t_n = ar^{n-1}$

Example 3

Find the sum of the first 5 terms of the geometric series 1 + 3 + 9 + 27 +...

Solution

Here, first term (a) = 1

Common ratio (r) = $\frac{3}{1} = \frac{9}{3} = 3$

Total number of terms (n) = 5

Here, r > 1, $S_n = \frac{a(r^n - 1)}{(r - 1)}$

So, $S_5 = \frac{1[(3^5 - 1)}{(3 - 1)}$

$= \frac{243 - 1}{2} = 121$

Hence, the sum of the first 5 terms of the series 1+3+9+27+…… is 121.

Example 4

Find the sum of the first 5 terms of the geometric series 2 + 1 + $\frac{1}{2}$ + $\frac{1}{4}$+...

Solution

Here, first term $(a) = 2$

Common ratio $(r) = \frac{1}{2}$

Number of terms $(n) = 5$

We know that $r < 1$, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

$$S_5 = \frac{2\{1-(\frac{1}{2})^5\}}{1-\frac{1}{2}} = \frac{2(1-(\frac{1}{32})}{\frac{1}{2}} = 4(1-\frac{1}{32}) = \frac{31}{8}$$

Hence, the sum of the first 5 terms of the geometric series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \ldots$ is $\frac{31}{8}$.

Example 5

Find the sum of the geometric series 3 + 6 + 12 + 24+ … + 768 .

Solution

Here, first term (a) = 2

Common ratio (r) = $\frac{6}{3} = 2$

Last term $(t_n) = 768$

Since the first term and the last term are given, $S_n = \frac{t_n r - a}{r - 1}$

$S_n = \frac{768 \times 2 - 3}{(2 - 1)} = 1536 - 3 = 1533$

Hence, the sum of geometric series 3 + 6 + 12 + 24 + … + 768 is 1533.

Example 6

Find the common ratio of a geometric series, if its first term is 7, last term is 448 and the sum of the terms is 889.

Solution

Here, first term $(a) = 7$

Last term $(l) = 448$

Total sum $(S_n) = 889$

The first and the last terms are given. So, $S_n = \frac{t_n r - a}{r - 1}$.

Therefore, $S_n = \frac{t_n r - a}{r - 1}$

or, $889 = \frac{448 \times r - 7}{r - 1}$

or, $889r - 889 = 448r - 7$

or, $889r - 448r = 889 - 7$

or, $441r = 882$

or, $r = \frac{882}{441} = 2$

Hence, the common ratio is 2.

Example 7

If the third and sixth terms of a geometric series are 27 and 729 respectively, find the sum of its first 10 terms.

Solution

Here, third term $(t_3) = 27$, sixth term $(t_6) = 729$

We know that $t_n = ar^{(n-1)}$

or, $t_3 = ar^{3-1} = ar^2$

or, $27 = ar^2$.....(i)

or, $t_6 = ar^{6-1} = ar^5$

or, $729 = ar^5$.....(ii)

Dividing the equation (ii) by (i)

or, $\frac{729}{27} = \frac{ar^2}{ar^2}$

or, $27 = r^3$

or, $3^3 = r^3$ or, $r = 3$

$\therefore r = 3$

Hence, the common ratio is 3.

Again, putting the value of r in equation (i),

$27 = a \times 3^2$

$\therefore a = 3$

Since $r > 1$, $S_n = \frac{a(r^n - 1)}{(r - 1)}$

or, $S_{10} = \frac{a\{(3)^{10} - 1\}}{(3 - 1)} = \frac{3(59049 - 1)}{2} = 88{,}572$

Hence, the sum of first 10 terms is 88,572.

Example 8

How many terms of a geometric series 64 + 32 + 16 +… will have a sum $\frac{255}{2}$?

Solution

Here, first term $(a) = 64$

Common ratio $(r) = \frac{32}{64} = \frac{1}{2}$

Sum of n terms $S_n = \frac{255}{2}$

Total number of terms $(n) = ?$

Here, $r < 1$, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

or, $\frac{255}{2} = \frac{64\{1 - (\frac{1}{2})^n\}}{(1 - \frac{1}{2})} = \frac{64\{1 - (\frac{1}{2})^n\}}{\frac{1}{2}} = 128\{1 - (\frac{1}{2})^n\}$

or, $\frac{255}{256} = 1 - (\frac{1}{2})^n$

or, $(\frac{1}{2})^n = 1 - \frac{255}{256}$

or, $(\frac{1}{2})^n = \frac{1}{256}$

or, $(\frac{1}{2})^n = (\frac{1}{2})^8$

$\therefore n = 8$

Hence, the number of terms to have a sum $\frac{255}{2}$ is 8.

Example 9

Hari borrowed Rs. 19,682 from Ram Naresh on condition that he would pay back in 9 installments. If the amount of each installment is three times of the former find the difference of first and last installments.

Solution

Here, common ratio $(r) = 3$ (3 times)

Sum of all the terms $(S_n) = 19682$

First term $(a) = ?$

Ninth term $(t_9) = ?$

Here, $r > 1$ So, $S_n = \frac{ar^n - 1}{r - 1}$

or, $19682 = \frac{a(3^9 - 1)}{(3 - 1)}$

or, $19682 = \frac{a(19683 - 1)}{2}$

or, $19682 = \frac{a \times 19682}{2}$

$\therefore a = 2$

The first installment = Rs. 2

Again, the last installment $(t_9) = ar^{n-1} = 2 \times 3^{9-1} = 2 \times 3^8 = 13122$

Then, the difference of the first and last installments is $13122 - 2$ = Rs. 13120.

Exercise 6.2

1. (a) What is meant by geometric mean?

(b) If the positive numbers a, m and b are in geometric sequence, express m in terms of a and b.

(c) What is the geometric mean between 3 and 27?

2. Find the geometric mean between the given two terms.

(a) –4 and –64 (b) $\frac{1}{5}$ and 125 (c) 7 and 343

3. Find the geometric mean in the following cases.

(a) 4 geometric means between 6 and 192

(b) 3 geometric means between 5 and 405

(c) 3 geometric means between $\frac{9}{4}$ and $\frac{4}{9}$

4. Find the value of x from the following geometric series.

(a) 4, x and 9

(b) x, 4, 8

(c) 5, 25 and $x+1$

5. Find the sum of the following geometric series.

(a) $2 + 4 + 8 + 16\ldots$, to 6 terms

(b) $\frac{1}{9} + \frac{1}{3} + 1 + \ldots$, to 5 terms

(c) $-\frac{1}{4} + \frac{1}{2} - 1 + \ldots$, to 6 terms

(d) $16 + 8 + 4 + \ldots, + \frac{1}{16}$

(e) $1 + \frac{1}{3} + \frac{1}{9} + \ldots, + \frac{1}{729}$

6. The first term, the last term and the sum of all the terms of geometric series are given below. Find their common ratio.

(a) First term = 2, last term = 486 and sum of all the terms = 728

(b) First term = 5, last term = 1215 and sum of all the terms = 1820

(c) First term = 3, last term = 768 and sum of all the terms = 1533

7. (a) Find the sum of the first 10 terms of a geometric series , if its second term is 4 and the seventh term is 128.

(b) Find the sum of the first 7 terms of a geometric series if its third term is 3 and the fifth term is 81.

8. (a) How many terms of a geometric series $32 + 48 + 72 + \ldots\ldots\ldots\ldots\ldots$will have a sum 665 ?

(b) How many terms of a geometric series $6 - 12 + 24 - 48 + \ldots\ldots$ will have a sum -2046 ?

9. Sarita borrowed Rs. 43680 from her friend Garima on condition that she would pay back it in 6 installments. If the amount of each installment she paid was three times of the previous one, find the difference of the first and the last installments.

Project Work

Form two groups A and B of your friends in order to solve the mathematical problems assigned by your teacher. Your teacher assigns questions to the different groups for 1 week as given below.

Group A: 3 questions for the first day, after then the number of questions for each day is two times of the previous day.

Group B: 1 question for the first day and the number of questions for each day is three times of the previous day.

Submit the work to your teacher after a week and find how many questions are solved by each group. Discuss the problem and its relation with sequence and series.

Answer

1. (b) $m = \sqrt{a \times b}$ (c) 9
2. (a) –16 (b) 5 (c) 49
3. (a) 12, 24, 48, 96 (b) 15, 45, 135 (c) $\frac{3}{2}, 1, \frac{2}{3}$
4. (a) 6 (b) 2 (c) 124
5. (a) 126 (b) $\frac{121}{9}$ (c) $\frac{21}{4}$
   (d) $31\frac{15}{16}$ (e) $1\frac{364}{729}$
6. (a) 3 (b) 3 (c) 2
7. (a) 2046 (b) 1093
8. (a) 6 (b) 10
9. Rs. 29,040