Lesson 7

Quadratic Equation

7.0 Review

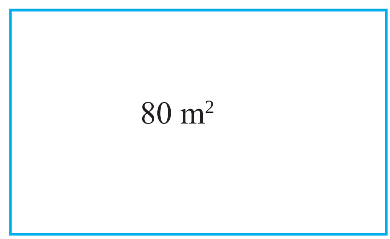
A school is carpeting its office room which is rectangular in shape and the amount of the carpet used is 80 m^2 . Discuss in pair and answer the following questions.

(a) What is the length and breadth of the office room?

(b) What is the length and breadth of the room if the length is greater than its breadth by 2 meter?

If the breadth of the room is x, then its length will be $= x + 2$

Area of the room $= 80 \text{ m}^2$



$(x + 2)x = 80$

or, $x^2 + 2x - 80 = 0$

or, $x^2 + 10x - 8x - 80 = 0$

or, $x(x + 10) - 8(x + 10) = 0$

or, $(x + 10)(x - 8) = 0$

either $x + 10 = 0$ $\quad \therefore x = -10$, which is not possible.

or, $x - 8 = 0$ $\quad \therefore x = 8$

Here, the length of the room $(l) = x + 2 = 8 + 2 = 10\ m$, breath of the room (b) $= x = 8$m

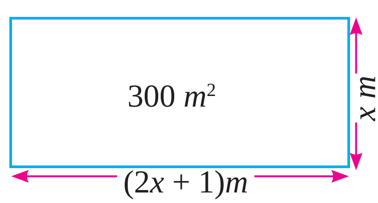
A quadratic equation is a second degree equation of one variable. It is in the form of $ax^2 + bx + c = 0$, where $a \neq 0$. There are two values of the variable satisfying the equation.

7.1 Solving Quadratic Equation

(a) Factorization Method

Activity 1

The area of a rectangular playground is 300 m^2. If its length is greater than breadth by 1 m, discuss in your group and find its length and breadth.



Here, area of a playground $= 300\ m^2$

If breadth of the playground $= x$

then, length of playground $= 2x + 1$

Now, area of rectangular playground = length × breadth

$300 = (2x + 1)\, x$

or, $2x^2 + x - 300 = 0$ [∵ This is quadratic equation.]

To find the value of x from above quadratic equation,

$2x^2 + (25 - 24)x - 300 = 0$

or, $2x^2 + 25x - 24x - 300 = 0$

or, $x(2x + 25) - 12(2x + 25) = 0$

or, $(2x + 25)(x - 12) = 0$

If multiplication of two factors is zero, then one factor should be zero.

either $(2x + 25) = 0$ or $(x - 12) = 0)$

If $2x + 25 = 0$

then, $2x = -25$

$x = -\frac{25}{2}$ is impossible

if, $x - 12 = 0$

or, $x = 12$ $\therefore x = 12$

Breadth of the playground (x) = 12 m, then length $= 2x + 1 = 2 \times 12 + 1 = 25\ m$

Example 1

Solve the equation and examine whether it is correct or not.

(a) $x^2 + 4x = 0$ (b) $x^2 + 6x + 8 = 0$ (c) $x^2 - 5x + 6 = 0$

(d) $x^2 - x - 6 = 0$ (e) $2x^2 + 7x + 6 = 0$

Solution

(a) $x^2 + 4x = 0$

or, $x(x + 4) = 0$

either $x = 0$

or, $x + 4 = 0$

$x = -4$

Thus, $x = 0, -4$

When examined,

$x = 0$ is putting in $x^2 + 4x = 0$

LHS $= 0 + 4 \times 0 = 0 =$ RHS

When placed $x = -4$

LHS $= (-4)^2 - 4 \times (-4) = 16 - 16 = 0 =$ RHS

(b) $x^2 + 6x + 8 = 0$

or, $x^2 + (4 + 2)x + 8 = 0$

or, $x^2 + 4x + 2x + 8 = 0$

or, $x(x + 4) + 2(x + 4) = 0$

or, $(x + 4)(x + 2) = 0$

either $x + 4 = 0$ $\therefore x = -4$

or, $(x + 2) = 0$ $\therefore x = -2$

When examined,

$x = -2$ putting $x^2 + 6x + 8 = 0$

LHS $= (-2)^2 + 6 \times (-2) + 80$

$4 - 12 + 8 = 0 =$ RHS

$x = -4$ is placed in $x^2 + 6x + 8 = 0$

LHS $= (-4)^2 + 6 \times (-4) + 8$

$= 16 - 24 + 8 = 0 =$ RHS

$\therefore$ The roots of quadratic equations are –2, –4

(c) $x^2 - 5x + 6 = 0$

or, $x^2 - (3 + 2)x + 6 = 0$

or, $x^2 - (3 + 2)x + 6 = 0$

or, $x^2 - 3x - 2x + 6 = 0$

or, $x(x - 3) - 2(x - 3)\ 0$

or, $(x - 3)(x - 2) = 0$

either, $(x - 3) = 0$ $\therefore x = 3$

or, $x - 2 = 0$ $\therefore x = 2$

When examined,

Putting $x = 2$

$x^2 - 5x + 6 = (2)^2 - 5 \times 2 + 6$

$= 4 - 10 + 6 = 0$

LHS = RHS

Again, when placed $x = 3$,

$(3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

LHS = RHS

$\therefore$ The roots of quadratic equation are 2 and 3.

(d) $x^2 - x - 6 = 0$

or, $x^2 - (3 - 2)\,x - 6 = 0$

or, $x^2 - 3x + 2x - 6 = 0$

or, $x(x - 3) + 2(x - 3) = 0$

or, $(x - 3)(x + 2) = 0$

either, $(x - 3) = 0$ $\therefore x = 3$

or, $x + 2 = 0$ $\therefore x = -2$

When examined,

Putting $x = 3$,

LHS $= (3)^2 - 3 - 6$

$= 9 - 9 = 0 =$ RHS

When placed $x = -2$

LHS $= (-2)^2 - 2 - 6$

$= 4 + 2 - 6 = 0 =$ RHS

$\therefore$ The roots of quadratic equation are 3 and -2.

(e) $2x^2 + 7x + 6 = 0$

or, $2x^2 + 7x + 6 = 0$

or, $2x^2 + (4 + 3)x + 6 = 0$

or, $2x^2 + 4x + 3x + 6 = 0$

or, $2x(x + 2) + 3(x + 2) = 0$

or, $(x + 2)(2x + 3) = 0$

either, $(x + 2) = 0$. $\therefore x = -2$

or, $2x + 3 = 0$. $\therefore x = -\frac{3}{2}$

$\therefore$ The roots of $2x^2 + 7x + 6 = 0$ are -2 and $-\frac{3}{2}$.

When examined

Putting $x = -2$

LHS $= 2(-2)^2 + 7 \times (-2) + 6$

$= 8 - 14 + 6 = 0 =$ RHS

Putting $x = -\frac{3}{2}$,

LHS $= 2 \times \left(-\frac{3}{2}\right)^2 + 7 \times -\frac{3}{2} + 6$

$= \frac{9}{2} - \frac{21}{2} + 6 = \frac{21 - 21}{2}$

$= 0 =$ RHS

(b) Solving quadratic equation by completing square

Activity 2

Solve the given quadratic equation

(a) $x^2 - 9 = 0$ (b) $x^2 - 5x + 6 = 0$

Solution

(a) $x^2 - 9 = 0$

or, $x^2 - 3^2 = 0$

or, $(x + 3)(x - 3) = 0$

Either $x + 3 = 0$ $\therefore x = -3$

or, $x - 3 = 0$ $\therefore x = 3$

$\therefore x = \pm 3$

We can also solve in this way.

$x^2 - 9 = 0$

or, $x^2 = 9$

or, $x^2 = 3^2$ [$\because$ Here x^2 and 9 both are square.]

or, $x = \pm 3$

The roots of $x^2 = a^2$ form of quadratic equation is $x \pm a$.

(b) $x^2 - 5x + 6 = 0$

or, $x^2 - 5x = -6$

or, $x^2 - 2\,\frac{5}{2}\,x + \left(\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2 - 6$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]

or, $\left(x - \frac{5}{2}\right)^2 = \frac{25}{4} - 6 = \frac{25 - 24}{4} = \frac{1}{4}$

or, $\left(x - \frac{5}{2}\right)^2 = \frac{1}{4}$

or, $\left(x - \frac{5}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

$\therefore x - \frac{5}{2} = \pm\frac{1}{2}$

Taking (+)ve sign,

$x - \frac{5}{2} = \frac{1}{2}$ or, $x = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$

Taking (-)ve sign,

$x - \frac{5}{2} = -\frac{1}{2}$

or, $x = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

Therefore, the roots of x is 2 and 3.

Example 2

Solve by completing the square

(a) $x^2 - 10x + 16 = 0$ (b) $x^2 - 7x + 12 = 0$ (c) $2x^2 - 7x + 6 = 0$

Solution

(a) $x^2 - 10x + 16 = 0$

or, $x^2 - 2 \times x \times 5 + (5)^2 - (5)^2 + 16 = 0$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]

or, $x^2 - 2 \times x \times 5 + (5)^2 - 25 + 16 = 0$

or, $(x - 5)^2 - 9 = 0$

or, $(x - 5)^2 = 9$

or, $(x - 5)^2 = 3^2$

or, $x - 5 = \pm 3$

Taking (+)ve sign,

$x - 5 = 3$ or, $x = 3 + 5 = 8$

Taking (-)ve sign,

$x - 5 = -3$ or, $x = 5 - 3 = 2$

$\therefore x = 8, 2$

(b) $x^2 - 7x + 12 = 0$

or, $x^2 - 2.\frac{7}{2}x + \left(\frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2 = 0$

or, $\left(x - \frac{7}{2}\right)^2 + 12 - \frac{49}{4} = 0$

or, $\left(x - \frac{7}{2}\right)^2 + \frac{48-49}{4} = 0$

or, $\left(x - \frac{7}{2}\right)^2 + \left(\frac{-1}{4}\right) = 0$

or, $\left(x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

or, $\left(x - \frac{7}{2}\right) = \pm\frac{1}{2}$

Taking (+)ve sign,

$x - \frac{7}{2} = \frac{1}{2}$ or, $x = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$

Taking (-)ve sign,

$x - \frac{7}{2} = -\frac{1}{2}$ or, $x = \frac{7}{2} - \frac{1}{2} = \frac{6}{2} = 3$

$\therefore$ x = 4, 3

(c) $2x^2 - 7x + 6 = 0$

or, $2x^2 - 7x + 6 = 0$

or, $2(x^2 - \frac{7}{2}x + 3) = 0$

or, $x^2 - \frac{7}{2}x + 3 = 0$

or, $x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + 3 = 0$

or, $x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 + 3 - \left(\frac{49}{16}\right) = 0$

or, $\left(x - \frac{7}{4}\right)^2 - \frac{1}{16} = 0$

or, $\left(x - \frac{7}{4}\right)^2 = \frac{1}{16}$

or, $\left(x - \frac{7}{4}\right)^2 = \left(\pm\frac{1}{4}\right)^2$

or, $\left(x - \frac{7}{4}\right) = \pm\frac{1}{4}$

Taking (+)ve sign,

$x - \frac{7}{4} = \frac{1}{4}$ or, $x = \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2$

Taking (-)ve sign, $x - \frac{7}{4} = -\frac{1}{4}$

or, $x = \frac{7}{4} - \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

$\therefore x = 2, \frac{3}{2}$

(C) Solving quadratic equation by using formula

Activity 3

How shall we find the value of quadratic equation $ax^2 + bx + c = 0$?

Here, $ax^2 + bx + c = 0$

or, $ax^2 + bx = -c$

or, $\frac{ax^2 + bx}{a} = -\frac{c}{a}$ [∴ dividing both sides by a]

or, $x^2 + \frac{bx}{a} = -\frac{c}{a}$

or, $x^2 + 2 \times x \frac{b}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = -\frac{c}{a}$ [∴ completing the square]

or, $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

or, $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$

or, $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

or, $\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2 - 4ac}{4a^2}\right)$

or, $x + \frac{b}{2a} = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$

or, $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

or, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Therefore, the roots of x are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Example 3

Solve the given quadratic equations by using formula.

(a) $x^2 - 5x + 6 = 0$ (b) $x\left(x - \frac{2}{7}\right) = \frac{3}{49}$

Solution

(a) Here, comparing $x^2 - 5x + 6 = 0$ to $ax^2 + bx + c = 0$, we get

$a = 1, b = -5, c = 6$

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4\times1\times6}}{2\times1}$$

$$= \frac{5 \pm \sqrt{25-24}}{2}$$

$$= \frac{5 \pm 1}{2}$$

Taking (+)ve sign, $x = \frac{5+1}{2} = \frac{6}{2} = 3$

Taking (-)ve sign, $x = \frac{5-1}{2} = \frac{4}{2} = 2$

Therefore, roots of x are 3 and 2.

(b) $x\left(x - \frac{2}{7}\right) = \frac{3}{49}$

Here, $x\left(x - \frac{2}{7}\right) = \frac{3}{49}$

or, $x^2 - \frac{2}{7}x - \frac{3}{49} = 0$

or, $49x^2 - 14x - 3 = 0$ Comparing $49x^2 - 14x - 3 = 0$ with

$ax^2 + bx + c = 0$ we get

$a = 49,\ b = -14,\ c = -3$

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4\times49\times(-3)}}{2\times49}$$

$$= \frac{14 \pm \sqrt{196+588}}{98}$$

Taking (+)ve sign, $x = \frac{14+28}{98} = \frac{42}{98} = \frac{3}{7}$

Taking (-)ve sign, $x = \frac{14-28}{98} = \frac{-14}{98} = -\frac{1}{7}$

Therefore, roots of x are: $\frac{3}{7}$ and $-\frac{1}{7}$

Exercise 7.1

1. Which of the following are the quadratic equation? Write with reason.

(a) $(x-2)^2 + 1 = 2x - 3$ (b) $x(x+1) + 8 = (x+2)(x-2)$

(c) $x(2x+3) = x^2 + 1$ (d) $(x+2)^3 = x^3 - 4$

(e) $x^2 + 3x + 1 = (x-2)^2$ (f) $(x+2)^3 = 2x(x^2 - 1)$

2. Solve by factorization method.

(a) $x^2 - 3x - 10 = 0$ (b) $2x^2 + x - 6 = 0$ (c) $2x^2 - x + \frac{1}{8} = 0$

(d) $100x^2 - 20x + 1 = 0$ (e) $x^2 - 45x + 324 = 0$ (f) $x^2 - 27x - 182 = 0$

3. Solve by completing square.

(a) $x^2 - 6x + 9 = 0$ (b) $9x^2 - 15x + 6 = 0$ (c) $2x^2 - 5x + 3 = 0$

(d) $5x^2 - 6x - 2 = 0$ (e) $x^2 + \frac{15}{16} = 2x$ (f) $x^2 + \frac{2}{3}x = \frac{35}{9}$

4. Solve by using formula.

(a) $x^2 - 9x + 20 = 0$ (b) $x^2 + 2x - 143 = 0$ (c) $3x^2 - 5x + 2 = 0$

(d) $2x^2 - 2\sqrt{2}\,x + 1 = 0$ (e) $x + \frac{1}{x} = 3$ (f) $\frac{1}{x} + \frac{1}{(x-2)} = 3,$

(g) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

5. Ramnaresh Mahato scored a total of 30 marks in two subjects VR. English and Mathematics in the first terminal examination of grade 10. If he scored 2 more marks in Mathematics and 3 fewer marks in English, then the product of his marks would be 210. Find his scores on both subjects.

6. A rectangular figure of a playground is given here. The length of the longer side of the playground is 30 m more than it's shorter side but its diagonal is 60 m more than its shorter side.

a) Find the length and breadth of the playground.

b) If 12 m × 3 m size artificial grass turfs have to be placed on the ground, how many turfs are needed?

c) If the ground has to be fenced 4 times with a barbed wire costing Rs. 15 per meter, how much will be the cost?

Answers

1. (a) Yes (b) no (c) yes (d) yes (e) no (f) no

2. (a) $5, -2$ (b) $-2, \frac{3}{2}$ (c) $\frac{1}{4}, \frac{1}{4}$ (d) $\frac{1}{10}, \frac{1}{10}$ (e) $9, 36$ (f) $13, 14$

3. (a) $3, 3$ (b) $1, \frac{2}{3}$ (c) $1, \frac{3}{2}$ (d) $\frac{3+\sqrt{19}}{5}, \frac{3-\sqrt{19}}{5}$
   (e) $\frac{3}{4}, \frac{5}{4}$ (f) $\frac{5}{3}, -\frac{7}{3}$

4. (a) $4, 5$ (b) $11, -13$ (c) $1, \frac{2}{3}$ (d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
   (e) $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ (f) $\frac{4+\sqrt{10}}{3}, \frac{4-\sqrt{10}}{3}$ (g) $1, 2$

5. 12, 18 or 13, 17

6. (a) 120 *m*, 90 *m* or 13, 17 (b) 300 (c) Rs. 25,200

7.2 Word problems related to quadratic equation

Activity 4

Age of Sumitra is 12 years and her sister is 18 years now. How many years after will the product of their age be 280? How can we find it?

	Sumitra's age	Sumitra's sister's age	Product of their ages
Now	12	18	216
1 year later	13	19	247
2 years later	14	20	280

Here,

Now, Sumitra's age = 12 years

Sumitra's Sister's age = 18 years

x years later,

Sumitra's age = $12 + x$

Sumitra's sister's age = $18 + x$

According to the condition given,

$(12 + x)(18 + x) = 280$

or, $216 + 18x + 12x + x^2 = 280$

or, $x^2 + 30x + 216 - 280 = 0$

or, $x^2 + 30x - 64 = 0$

or, $x^2 + 32x - 2x - 64 = 0$

or $x(x + 32) - 2(x + 32) = 0$

or, $(x + 32)(x - 2) = 0$

Either $x + 32 = 0$ $\quad \therefore x = -32$

Or, $x - 2 = 0$ $\quad \therefore x = 2$

Here, $x = -32$ is not a possible solution because age cannot be negative.

Therefore, $x = 2$

2 years later, the product of their age will be 280.

Example 4

If the sum of two positive numbers is 18 and their product is 77, then find these numbers.

Solution

Let these two numbers be x and y.

According to the question,

$x + y = 18$............ (i)

$x \times y = 77$............ (ii)

From equation (i) $y = 18 - x$........(iii)

Placing the value of y in equation (ii)

$x(18 - x) = 77$

or, $18x - x^2 = 77$

or, $18x - x^2 - 77 = 0$

or, $x^2 - 18x + 77 = 0$

or, $x^2 - 11x - 7x + 77 = 0$

or, $x(x - 11) - 7(x - 11) = 0$

or, $(x - 11)(x - 7) = 0$

Either, $(x - 11) = 0 \quad \therefore x = 11$

Or, $(x - 7) = 0 \quad \therefore x = 7$

Placing the value of x in equation (iii),

If $x = 11$ then $y = 18 - x = 18 - 11 = 7$

If $x = 7$ then $y = 18 - x = 18 - 7 = 11$

Therefore, two positive numbers are 7 and 11 or 11 and 7.

Example 5

If 11 is sbutracted from the square of a positive integer, then the result is 38. Find the number.

Solution

Let the positive integer be x, then its square is x^2.

According to the question, $x^2 - 11 = 38$

or, $x^2 - 11 = 38$

or, $x^2 = 38 + 11$

or, $x^2 = 49$

or, $x^2 = (\pm 7)^2$

$\therefore x = \pm 7$

But we need a positive integer, so x = 7 only.

Therefore, 7 is the required positive integer.

Example 6

If the product of two consecutive positive even numbers is 24, then find these numbers.

Solution

Let, the two consecutive even numbers be x and $x + 2$.

According to the question,

$x \times (x + 2) = 24$

or, $x^2 + 2x - 24 = 0$

or, $x^2 + 6x - 4x - 24 = 0$

or, $x(x + 6) - 4(x + 6) = 0$

or, $(x + 6)(x - 4) = 0$

Either, $(x + 6) = 0$ $\therefore x = -6$ [$\because$ This is a negative number]

Or, $x - 4 = 0$ $\therefore x = 4$

Therefore, the required two positive numbers are 4 and 4 + 2 = 6.

Example 7

If the sum of a number and its reciprocal is $\frac{26}{5}$ then find the number.

Solution

Let, the number be x and the reciprocal of that number be $\frac{1}{x}$

According to the question,

$x + \frac{1}{x} = \frac{26}{5}$

or, $\frac{x^2 + 1}{x} = \frac{26}{5}$

or, $5x^2 + 5 = 26x$

or, $5x^2 - 26x + 5 = 0$

or, $5x^2 - 25x - x + 5 = 0$

or, $5x(x-5) - 1(x-5) = 0$

or, $(5x-1)(x-5) = 0$

Either, $(5x-1) = 0$ $\quad\therefore x = \frac{1}{5}$

Or, $x - 5 = 0$ $\quad\therefore x = 5$

Therefore, the required numbers are 5 and $\frac{1}{5}$

Example 8

The sum of the two brothers' age is 34 and the product of their ages is 288, then find their present age.

Solution

Let the age of the elder brother and younger brother be x and y respectively.

According to to question,

$x + y = 34$............ (i)

$x \times y = 288$............ (ii)

From equation (i) $y = 34 - x$........(iii)

Placing the value of y in equation (ii)

or, $x(34 - x) = 288$

or, $34x - x^2 = 288$

or, $x^2 - 34x + 288 = 0$

or, $x^2 - 16x - 18x + 288 = 0$

or, $x(x - 16) - 18(x - 16) = 0$

or, $(x - 16)(x - 18) = 0$

Either, $x - 16 = 0$ $\quad\therefore x = 16$

Or, $x - 18 = 0$ $\quad\therefore x = 18$

Placing the value of x in equation (iii)

If $x = 16$ then $y = 34 - x = 34 - 16 = 18$ [$\because$ This is not possible.]

If $x = 18$ then $y = 34 - x = 34 - 18 = 16$

Therefore, the age of elder brother is 18 and the age of younger brother is 16.

Example 9

The product of the digits of a two digit number is 18. If 27 is added to the number, the places of digits are reversed. What is the number? Find it.

Solution

Let, the two digit number $= 10x + y$ [∵ where x is ten place and y is once place digit.]

According to the question,

$x + y = 18$

or, $x = \frac{18}{y}$ .... (i)

Again the second condition, $(10x + y) + 27 = 10y + x$

$10x + y + 27 - 10y - x = 0$

or, $9x - 9y + 27 = 0$

or, $9(x - y + 3) = 0$

or, $x - y + 3 = 0$ .... (ii)

Placing the value of x from equation (i) to equation (ii)

or, $\frac{18}{y} - y + 3 = 0$

or, $\frac{18 - y^2 + 3y}{y} = 0$

or, $y^2 - 3y - 18 = 0$

or, $y^2 - 6y + 3y - 18 = 0$

or, $y(y - 6) + 3(y - 6) = 0$

or, $(y - 6)(y + 3) = 0$

Either $y - 6 = 0$ $\therefore y = 6$

Or, $y + 3 = 0$ $\therefore y = -3$

Placing the value of y in equation (ii)

If $y = 6$, then $x = \frac{18}{6} = 3$.

If $y = -3$, then $x = \frac{18}{-3} = -6$.

If $y = 6$ and $x = 3$ then the number is $10x + y = 10x\ 3 + 6 = 36$.

If $y = -3$ and $x = -6$ then the number is $10x + y = 10x\ (-6) - 3 = -63$.

Example 10

The present age of the father and his son is 42 years and 16 years respectively. Find how many years ago the product of their age was 272.

Solution

Let, x years ago, the age of the father and his son was $42 - x$ and $16 - x$ respectively.

According to the question,

x years ago, the product of their age = 272.

or, $(42 - x)(16 - x) = 272$

or, $672 - 42x - 16x + x^2 = 272$

or, $x^2 - 58x + 400 = 0$

or, $x^2 - 50x - 8x + 400 = 0$

or, $x(x - 50) - 8(x - 50) = 0$

or, $(x - 8)(x - 50) = 0$

Either, $x - 8 = 0$ $\quad\quad \therefore x = 8$

O, $x - 50 = 0$ $\quad\quad \therefore x = 50$

Here, $x = 50$ years, which is impossible. Therefore, $x = 8$.

Hence, 8 years ago the product of the father and his son's age was 272.

Example 11

The length of the hypotenuse of a right angled triangle is 13m. If the difference of its other two sides is 7m, find the length of the remaining sides.

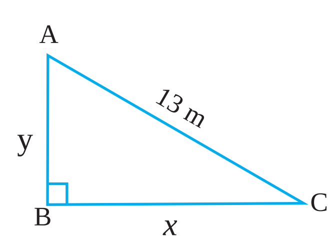
Solution

Here, ΔABC is a right angled triangle where $\angle B = 90^0$ and hypotenuse (h) = AC = 13m

Let base (b) = BC = x and perpendicular (p) = AB = y.

According to the question,

$x - y = 7$ $\quad\quad$ or, $y = x + 7$..........(i)



Now, in the right angled triangle ABC $h^2 = p^2 + b^2$.

Therefore, $AC^2 = AB^2 + BC^2$

or, $13^2 = (x + 7)^2 + x^2$

or, $169 = x^2 + 14x + 49 + x^2$

or, $2x^2 + 14x – 120 = 0$

or, $x^2 + 7x – 60 = 0$

or, $x^2 + 12x – 5x – 60 = 0$

or, $x (x + 12) – 5(x +12) = 0$

or, $(x – 5) (x + 12) = 0$

Either, $(x – 5) = 0$ $\therefore\ x = 5$

Or, $x + 12 = 0$ $\therefore\ x = 0$

Here, x = -12 is impossible because length of a base cannot be negative, therefore $x = 5$.

Hence, base (b) = BC = x = 5m

and perpendicular (p) = AB = y = 5 + 7 = 12m.

Hence, the remaining sides are 5m and 12m.

Example 12

The area of a rectangular land is 50m² and its perimeter is 90m. If the land is to be made square, by what percentage should it be reduced in length?

Solution

Let, the length and breadth of the rectangular land are x and y respectively.

According to question,

Area of a the rectangular land = $50m^2$

or, xy = 500...................(i)

Perimeter of the rectangular land = 90m

or, $2(x + y) = 90$

or, $x + y = 45$

or, $y = 45 – x$ ....(ii)

Now, place y = 45 – x from equation (ii) in equation (i).

$xy = 500$

or, $x(45 - x) = 500$

or, $45x - x^2 = 500$

or, $x^2 - 45x + 500 = 0$

or, $x^2 - 25x - 20x + 500 = 0$

or, $x(x - 25) - 20(x - 25) = 0$

or, $(x - 25)(x - 20) = 0$

either, $(x - 25) = 0$. $\therefore x = 25$

or, $x - 20 = 0$ $\therefore x = 20$

If $x = 25$, $y = 45 - x = 45 - 25 = 20$.

If $x = 20$, $y = 45 - x = 45 - 20 = 25$.

Therefore, the length and breadth of the rectangular land are 25m and 20m respectively.

If the land is to be made square, then the length and breadth of the land should be equal.

So, the length should be reduced by 5m.

Therefore, percentage to be reduced in length = $\frac{5}{25} \times 100\% = 20\%$

Example 13

Some students of grade 10 organized a picnic of Rs. 42,000 budget. They decided to collect equal money for picnic. But, 5 students were absent in the picnic day, so each student should collect Rs. 700 more. Based on this context, solve the problem given below.

a) How many students attended the picnic?

b) How much did each participant have to pay? Find it.

Solution

Let, the number of students = x and the amount to be paid by each participant = Rs. $\frac{42000}{x}$

Here, 5 students were absent.

Therefore, the number of students participated = $x - 5$

According to the question,

$$\frac{42000}{x-5} = \frac{42000}{x} + 700$$

or, $\frac{42000}{x-5} - \frac{42000}{x} = 700$

or, $\frac{60}{x-5} - \frac{60}{x} = 1$

or, $60x - 60x + 300 = x(x-5)$

or, $x^2 - 5x - 300 = 0$

or, $x^2 - 20x + 15x - 300 = 0$

or, $x(x-20) + 15(x-20) = 0$

or, $(x-20)(x+15) = 0$

Either, $(x-20) = 0$ $\therefore x = 20$

Or, $(x+15) = 0$ $\therefore x = -15$

Here, x denotes the number of students, so $x = -15$ is impossible. Therefore, $x = 20$.

Hence,

a) The total number of students participated in picnic = 20 – 5 = 15

b) The total amount of money per person = $\frac{42000}{x-5} = \frac{42000}{15}$ = Rs. 2800

Exercise 7.2

1. If 11 is added to the square of a natural number, the sum is 36. Find the number.
2. If 11 is subtracted from the square of a number, the remainder number is 25. Find the number.
3. If 7 is subtracted from the double of a square of a positive number, the remainder is 91. Find the number.
4. If 2 is subtracted from the square of a natural number, the remainder is 7. Find the number.
5. If 11 is subtracted from the square of a number and the remainder is 89, find that number.
6. If 17 is subtracted from the square of a number, the remainder is 55. Find the number.
7. If 3 is subtracted from the double ofthe square of a positive number, the remainder is 285. Find the number.

8. If the sum of a number and its square is 72, find the number.
9. If the product of the two consecutives even numbers is 80, find the numbers.
10. If the product of the two consecutive odd numbers is 225, find the numbers.
11. If the sum of a number and their reciprocal is $\frac{10}{3}$, find the number.
12. If the sum of two natural numbers is 21 and sum of their square is 261, find the numbers.
13. If the age difference between two brothers is 4 years and the product of their ages is 221. Find their age.
14. The sum of the present age of two brothers is 22 and the product of their ages is 120. Find their present age.
15. The age difference between two sisters is 3 years and the product of their age is 180. Find their present age.

16. (a) The present age of a father and his son is 40 years and 13 years respectively. Find how many years ago the product of their age was 198.

(b) The present age of a mother and her daughter is 34 years and 4 years respectively. Find how many years later the product of their age will be 400.

(c) The present age of a father and his son is 35 years and 1 year respectively. Find how many years later the product of their age will be 240.

(d) The present age of a husband and wife is 35 years and 27 years respectively. Find how many years ago the product of their age was 425.

17. (a) The hypotenuse of a right angled triangle is 25m. If the difference between its other two sides is 17m, find the length of the remaining sides.

(b) The length of hypotenuse of a right angled triangle is double and 6m more than its shortest side. If the length of the remaining side is 2m less than hypotenuse, find the length of all sides.

(c) Calculate the length and breadth of a rectangular land whose area is $150m^2$ and perimeter is 50m.

(d) Calculate the length and breadth of a rectangular land whose area is $54m^2$ and perimeter is 30m.

(e) Calculate the area of a rectangle whose length is 24m and diagonal is 16m more than its breadth.

(f) The area and perimeter of a rectangular land is $2000m^2$ and 180m respectively. If the land is to be made square, by what percentage should it be reduced in length or breadth? Calculate.

18. The two digit number is equal to the four times the sum of their digits and three times the product of their digits. Find the number.

19. An institute made a plan to distribute 180 pencils equally to the students enrolled in grade one. On the pencil distribution day, 5 students were absent so that each students got 3 more pencils.

    (a) How many students enrolled in grade one?

    (b) How many pencil did each student receive in total?

Project work

Form three groups of students for preparing a volleyball court of your school. The first group made a volleyball court whose area is $128m^2$ and perimeter is 48m. The second group made a volleyball court whose area is $162m^2$ and perimeter is 54m. The third group made a volleyball court whose area is $200m^2$ and perimeter is 60m. Discuss the dimension of volleyball court in group and conclude which court is suitable for playing volleyball. Present your conclusion in your class.

Answers

1. 5 2. ±6 3. 7 4. 3 5. ±10 6. ±6

7. 12 8. 8 9. 8 and 10 or –10 and –8

10. 3 and 5 or –5 and –3 11. 3 and 1/3 12. 6 and 15

13. 17 years and 13 years 14. 12 years and 10 years 15. 15 years and 12 years

16. (a) 7 years (b) 6 years (c) 5 years (d) 10 years

17. (a) 24 m and 7m (b) 10m, 24m, 26m (c) 15 m and 10 m

(d) 9 m and 6 m (e) 240 m^2 (f) less then 20%

18. 24 19. (a) 20 person (b) 12 pencil